



# **Accounting for polarisation effects in bio-molecular simulation**

# Inclusion of polarisability in molecular models

**A. Why polarisability ?**

**B. Six aspects**

**C. Charge-on-spring (COS) models**

**D. Representation of entropy**

# Inclusion of polarisability in molecular models

## A. Why polarisability ?

### 1. Effects of polarisability

## B. Six aspects

## C. Charge-on-spring (COS) models

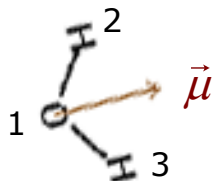
## D. Representation of entropy

# Effects of polarisability

## I Liquids, water

### 1. Dipole moment of a water molecule:

$$\vec{\mu} = \sum_{i=1}^3 q_i \vec{r}_i$$



gas phase = 1.85 Debye  
liquid phase  $\sim$  2.5 Debye

**water molecule is polarised  
when immersed in water**

### 2. Polarisation energy of liquid water

potential energy:  $\sim 10$  kJ/mol  
42 kJ/mol

**polarisation energy  $\sim$  25%**

$$1 \text{ eV} = 97 \text{ kJ/mol}$$

$$\text{or } 1 \text{ kJ/mol} = 0.01 \text{ eV}$$

$$1 \text{ kcal/mol} = 4.184 \text{ kJ/mol} \quad \text{or} \quad 1 \text{ kJ/mol} = 0.24 \text{ kcal/mol}$$

# Effects of polarisability

## 3. Macroscopic polarisation as function of the frequency $\omega$ of the oscillating electric field $\vec{E}$

the dielectric permittivity  $\epsilon_r(\omega)$

is dependent on the frequency  $\omega$   
of the oscillating field  $\vec{E}(\omega)$

liquid water  $\epsilon_r(0) = \epsilon_r(\text{static}) = 78$       electronic + orientation

$\epsilon_r(\infty) = \epsilon_r(\text{infinite freq.}) \approx 2-5$       only electronic

**Consequence: gas phase water models**

(e.g. derived using quantum-mechanical models)

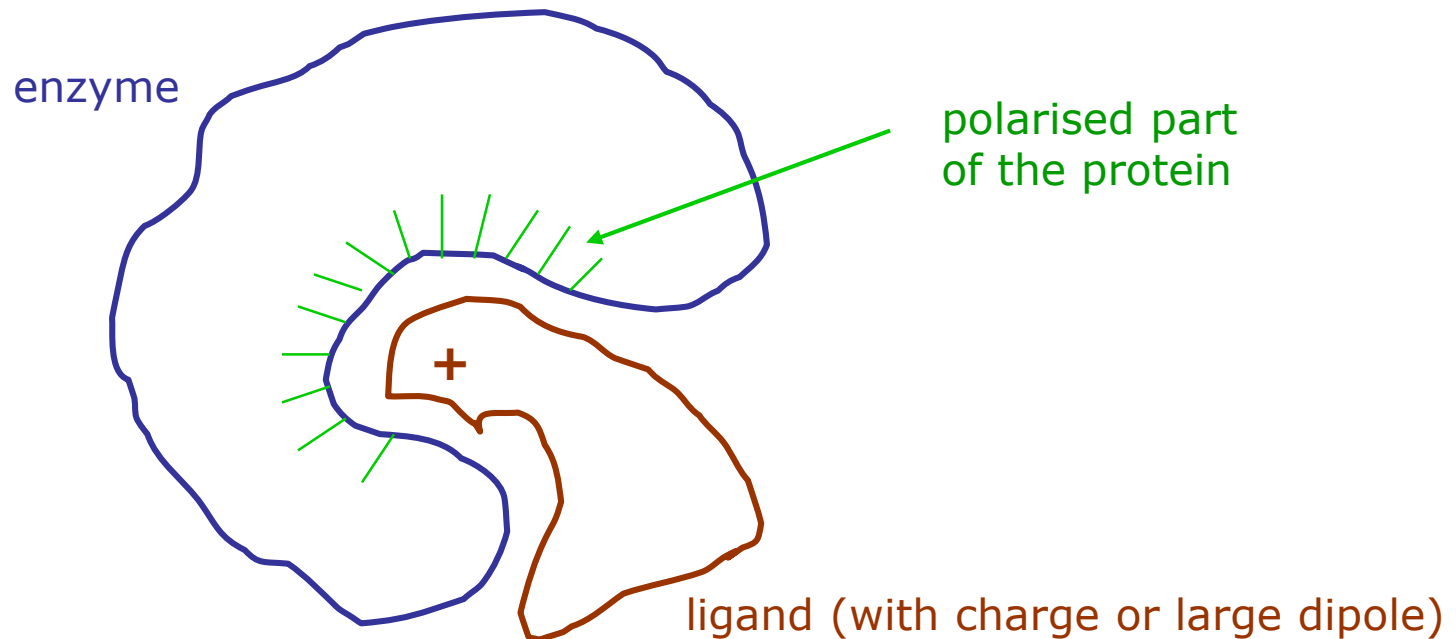
**will *not* represent liquid water,  
if explicit polarisation is not included**

# Effects of polarisability

## II Lipids and proteins in membranes

Carbon atoms of lipids can get polarised upon protein insertion

## III Proteins, ligand binding



Protein can get polarised upon ligand binding

# Inclusion of polarisability in molecular models

## A. Why polarisability ?

1. Effects of polarisability
2. Limitations of non-polarisable force fields

## B. Six aspects

## C. Charge-on-spring (COS) models

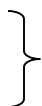
## D. Representation of entropy

# Limitations of non-polarisable force fields

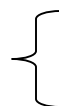
## 1. Solvation of polar amino-acid side-chain analogs

One set of atomic charges and van der Waals parameters cannot reproduce both:

heat of vaporisation  
of the liquid



and



free energy of solvation in  
polar solvent: water

GROMOS **53A5** parameters

GROMOS **53A6** parameters

due to the different polarisability of these two environments.

Free energy of solvation in apolar solvent, cyclohexane, is compatible with both using a non-polarisable force field.

## 2. Solvation of dimethylether (DME)

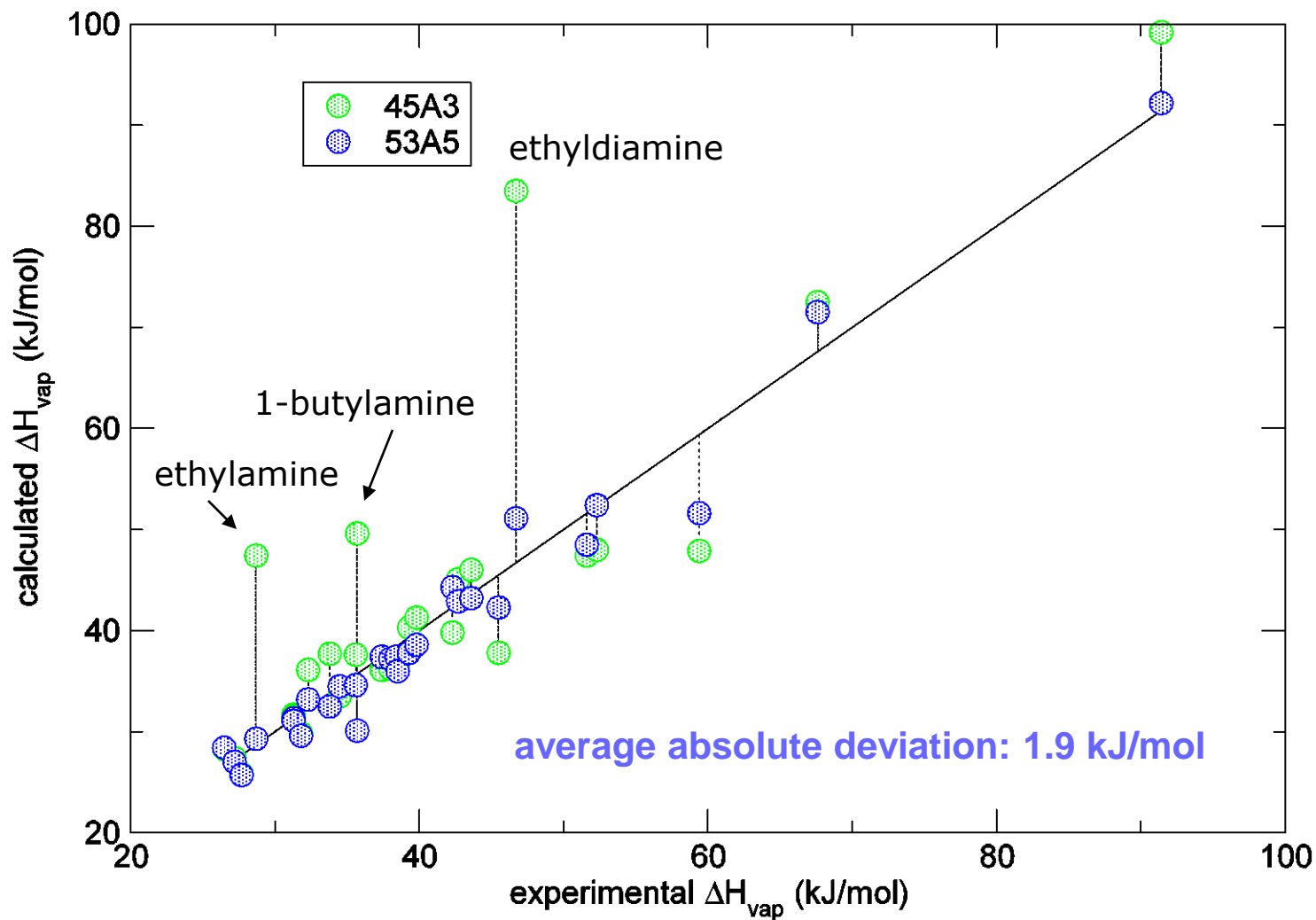
Polarisable models yield better (free) energies (of solvation)

## 3. Hydrophobic solvation in water – ethylene glycol mixtures

Polarisable models can reproduce the maximum in the free enthalpy of solvation as function of ethylene glycol mole fraction



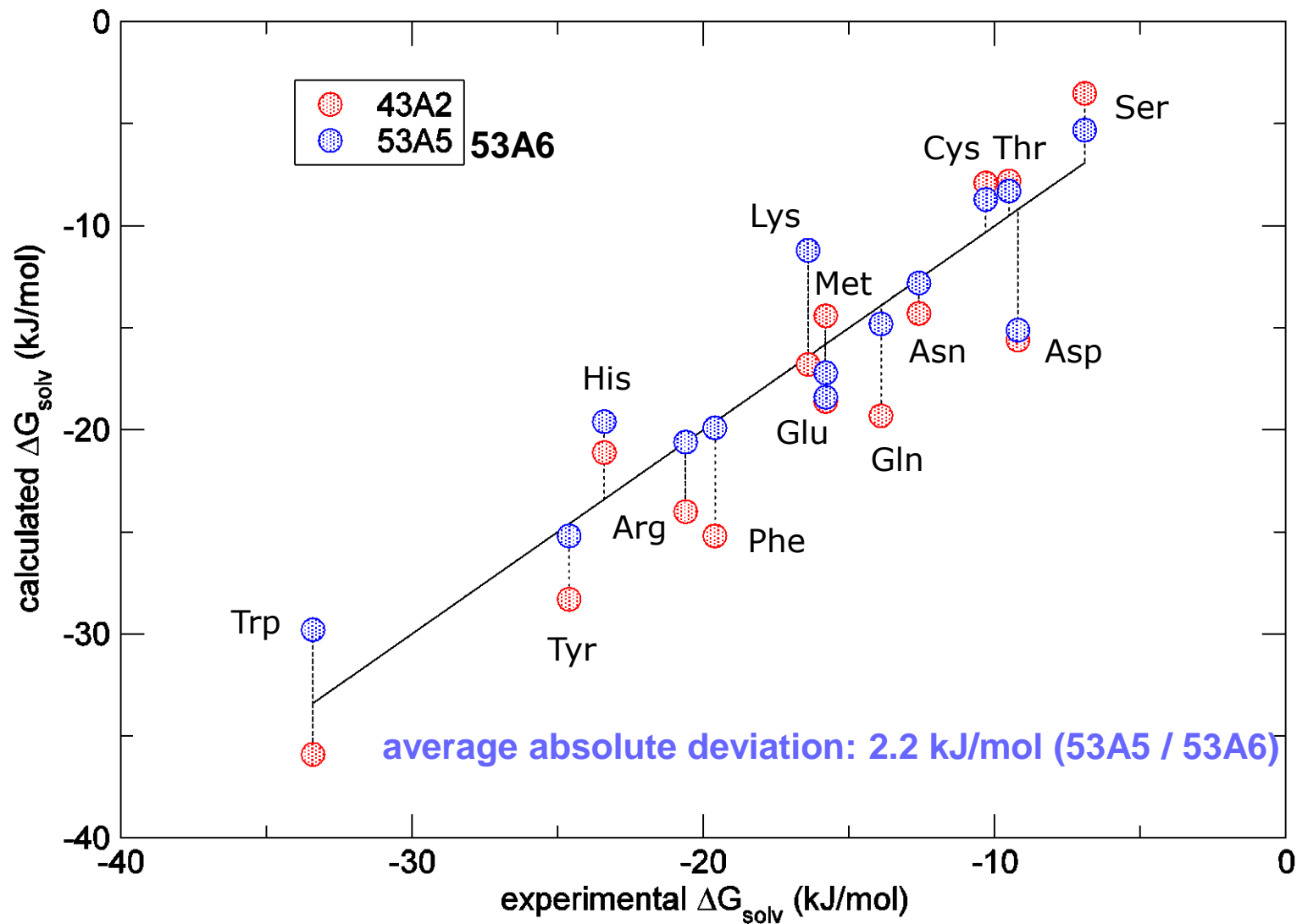
# Heat of vaporization for pure liquids



Oostenbrink et al., *J Comput. Chem.* 25 (**2004**) 1656-1676

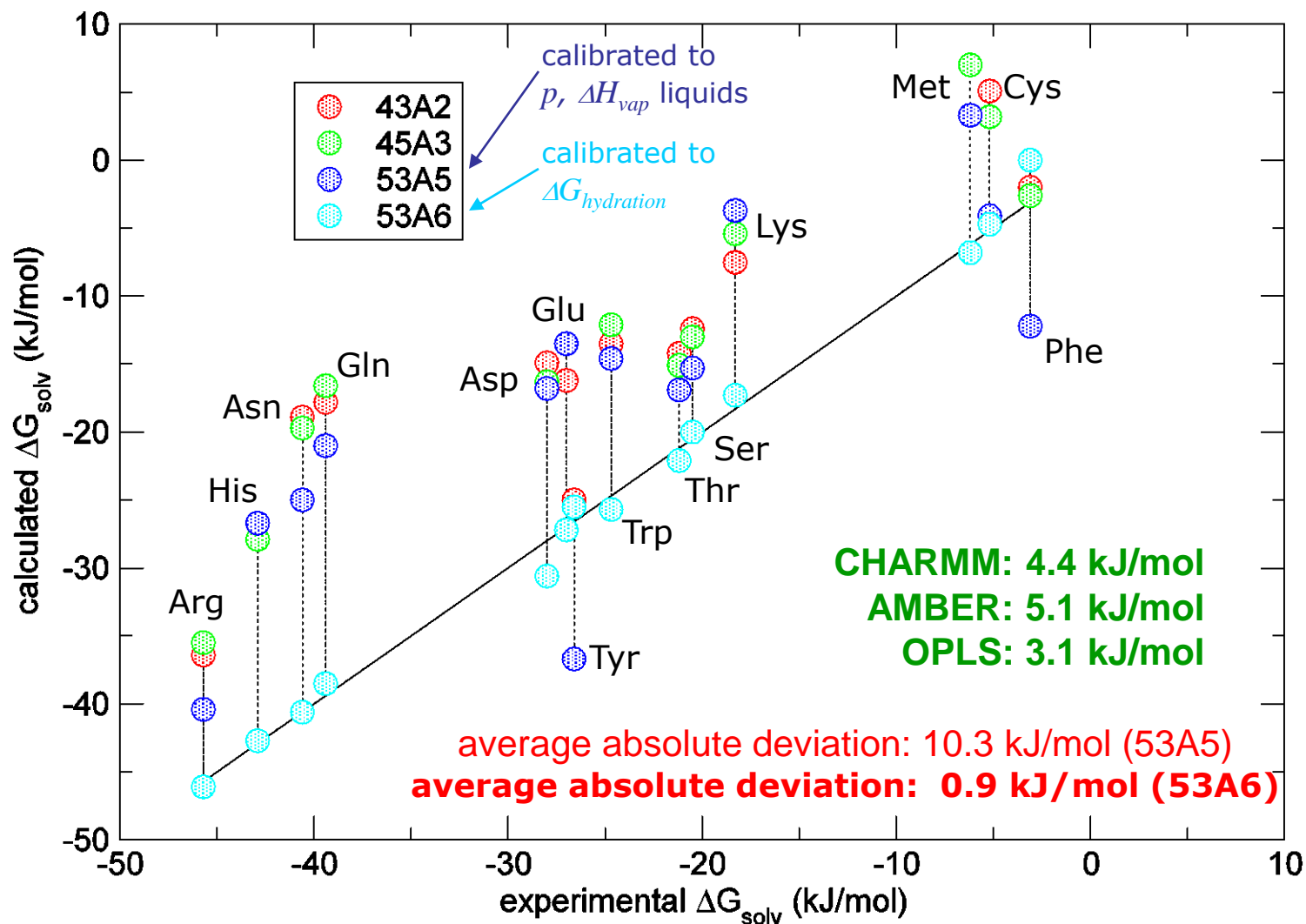
# Free energy of solvation in cyclohexane

amino acid analogues (polar)



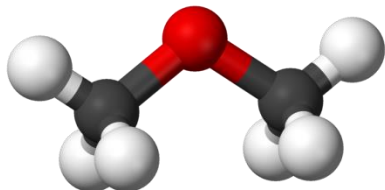
# Free energy of solvation in water

## amino acid analogues (polar)



# Use of polarisable force fields

A **polarisable** force field for **dimethyl ether**, **cyclohexane** and **water**



	<i>pure</i> DME	1 DME in <i>cyclohexane</i>	1 DME in <i>water</i>
	$\Delta H_{\text{vap}}$ [kJ mol <sup>-1</sup> ]	$\Delta G_{\text{solv}}$ [kJ mol <sup>-1</sup> ]	$\Delta G_{\text{hydr}}$ [kJ mol <sup>-1</sup> ]
Experiment	21.7	~ -9.5	~ -7.6
DME <sub>nonpol</sub> /solv <sub>nonpol</sub>	21.3	-9.3	-3.2
DME <sub>pol</sub> /solv <sub>nonpol</sub>	21.0	-8.3	-9.6
DME <sub>pol</sub> /solv <sub>pol</sub>	21.0	-9.9	-7.0

**Non-polarisable force field** has difficulty reproducing solvation in *non-polar* and *polar* solvent.

**Polarisable DME** improves the solvation in *water*, but degrades solvation in *cyclohexane*.

**Polarisable solvent** increases the **transferability** between different environments further.

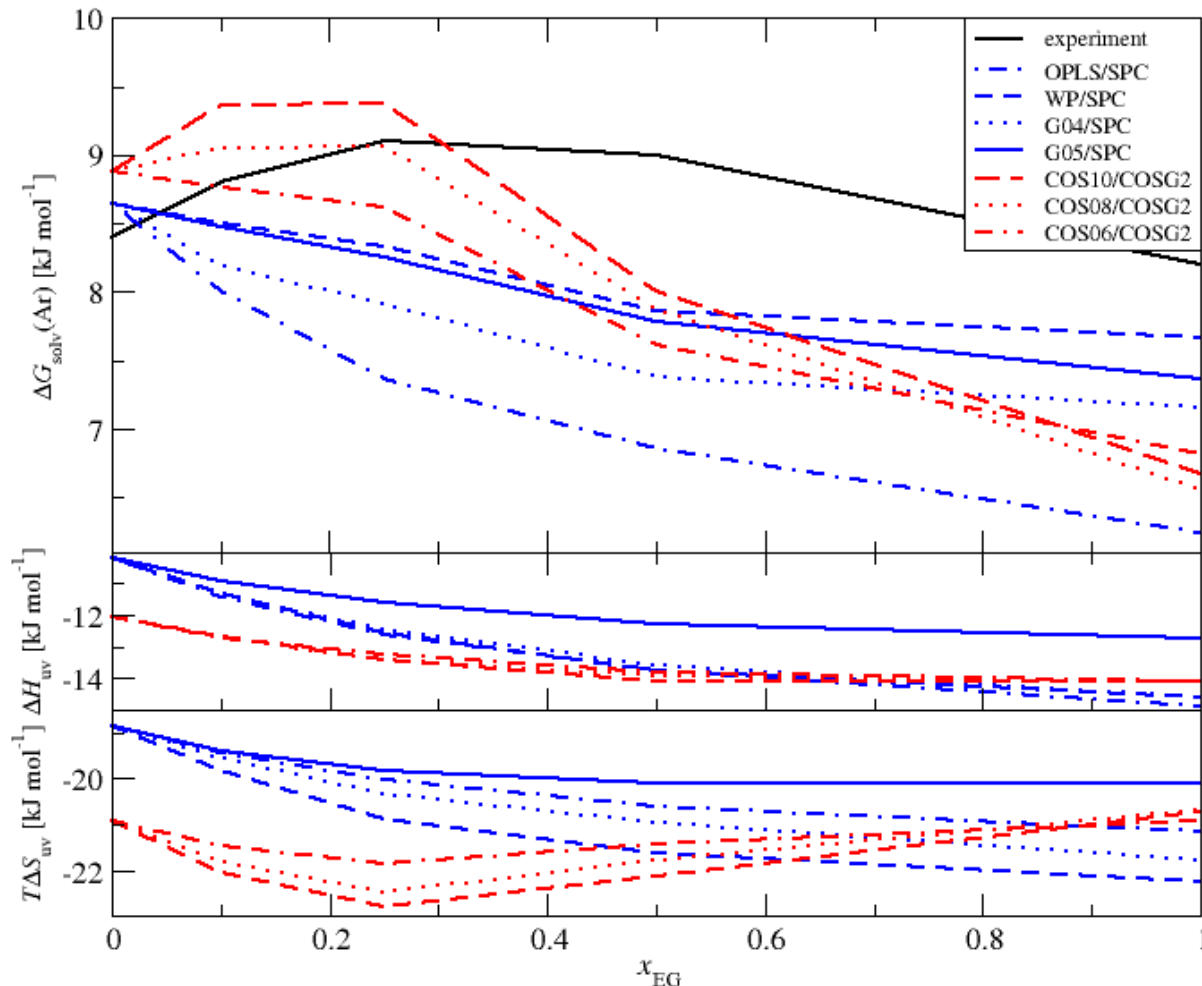
# Hydrophobic hydration (argon) in water-ethylene-glycol mixtures

Non-polarisable models versus polarisable models

Free enthalpy  
of solvation

Solute-solvent  
enthalpy  
of solvation

Solute-solvent  
entropy  
of solvation



Only polarisable models reproduce the experimental trend

# Inclusion of polarisability in molecular models

## A. Why polarisability ?

1. Effects of polarisability
2. Limitations of non-polarisable force fields

## B. Six aspects

1. Definition, distinctions, approximations

## C. Charge-on-spring (COS) models

## D. Representation of entropy

# Accounting for polarisation in molecular simulation

## Polarisation

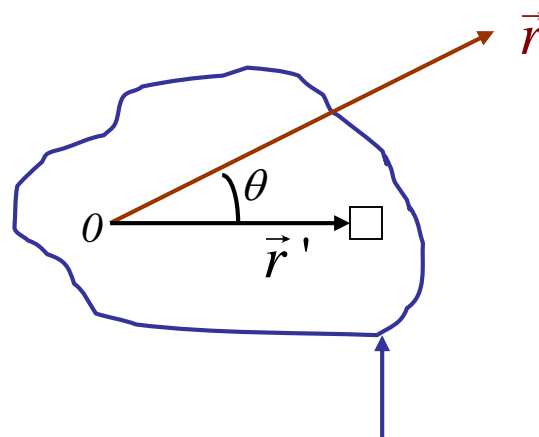
Change the *multipole moments*  $\mu^{(n)}$  of a charge distribution due to an (applied) *electric field*  $\vec{E}$

monopole:	charge	$q$	} of charge distribution
dipole:	dipole	$\vec{\mu}$	
quadrupole:	...		
octupole:	...		

# Potential and field of a charge distribution

## Coulomb potential

$$\varphi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}'$$



## Series expansion of the integrand

$$\frac{1}{|\vec{r} - \vec{r}'|} = \left[ (x - x')^2 + (y - y')^2 + (z - z')^2 \right]^{-\frac{1}{2}}$$

$$= \left[ x^2 + y^2 + z^2 - 2(xx' + yy' + zz') + x'^2 + y'^2 + z'^2 \right]^{-\frac{1}{2}}$$

$$= \left[ r^2 - 2\vec{r} \cdot \vec{r}' + r'^2 \right]^{-\frac{1}{2}}$$

$$= \frac{1}{r} \left[ 1 - \frac{2\vec{r} \cdot \vec{r}'}{r^2} + \left( \frac{r'}{r} \right)^2 \right]^{-\frac{1}{2}}$$

$$= \text{assume } |\vec{r}'| \ll |\vec{r}| \text{ and use } [1 + x]^{-\frac{1}{2}} = 1 - \frac{1}{2}x + \frac{3}{8}x^2 - \frac{5}{16}x^3 + \dots$$



# Potential and field of a charge distribution

## Series expansion of the integrand

$$\frac{1}{|\vec{r} - \vec{r}'|} = \frac{1}{r} \left[ 1 + \frac{\vec{r} \cdot \vec{r}'}{r^2} - \underbrace{\frac{1}{2} \left( \frac{r'}{r} \right)^2 + \frac{3}{2} \frac{(\vec{r} \cdot \vec{r}')^2}{r^4}}_{\text{III}} - \frac{3}{2} \frac{(\vec{r} \cdot \vec{r}') \vec{r}'^2}{r^4} + \frac{3}{8} \left( \frac{r'}{r} \right)^4 \dots \right]$$

**I**   **II**

1    $\frac{r'}{r}$

**III**

$\left( \frac{r'}{r} \right)^2$

**IV**

$\left( \frac{r'}{r} \right)^3$

Term I:    monopole     $\varphi^I(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r} \int \rho(\vec{r}') d\vec{r}' = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$      $q = \text{charge}$

Term II:    dipole     $\varphi^{II}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{\vec{r}}{r^3} \cdot \int \vec{r}' \rho(\vec{r}') d\vec{r}' = \frac{1}{4\pi\epsilon_0} \frac{\vec{\mu} \cdot \vec{r}}{r^3}$      $\mu = \text{dipole-moment}$

Term III:    quadrupole     $\varphi^{III}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \int \frac{1}{2} [3\cos^2\theta - 1] r'^2 \rho(\vec{r}') d\vec{r}'$

$= \frac{1}{4\pi\epsilon_0} \frac{Q(\theta)}{r^3}$      $Q = \text{quadrupolemoment}$

# Potential and field of a charge distribution

## Electric field

differentiate  $\varphi$  with respect to  $\vec{r}$ , take minus sign:

$$\text{Term I: } E^I(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \frac{\vec{r}}{r} = \frac{q\vec{r}}{4\pi\epsilon_0 r^3}$$

$$\text{Term II: } E^{II}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[ -\frac{\vec{\mu}}{r^3} + \frac{3\vec{r}(\vec{\mu} \cdot \vec{r})}{r^5} \right] = \underbrace{\frac{1}{4\pi\epsilon_0} \frac{1}{r^3} \left[ \frac{3\vec{r}\vec{r}}{r^2} - 1 \right]}_{\text{dipole tensor } \vec{T}} \cdot \vec{\mu}$$

dipole tensor  $\vec{T}$

# Macroscopic description of polarisation

Continuum with relative dielectric permittivity  $\epsilon_r$   
or susceptibility  $\chi$

- **dielectric displacement:**  $\vec{D} = \epsilon_0 \epsilon_r \vec{E}$

- **polarisation:**  $\vec{P} = (\epsilon_r - 1) \epsilon_0 \vec{E} = \chi \vec{E}$

- **Coulomb energy:** 
$$U(r_i, r_j) = \frac{1}{4\pi\epsilon_0 \epsilon_r} \frac{q_i q_j}{|r_i - r_j|}$$

# Accounting for polarisation in molecular simulation

## Distinguish:

### 1. **Dipole** moment versus other **multipole** moments

- monopole generally does not change (charge conservation)
- dipole term then dominates expansion

### 2. **Homogeneous** versus **inhomogeneous** fields

$$\vec{E} = \vec{E}(\vec{r}) \quad \vec{E} \text{ varies with } \vec{r}$$

### 3. **Linear** versus **non-linear** dependence of induced moment on field $\vec{E}$

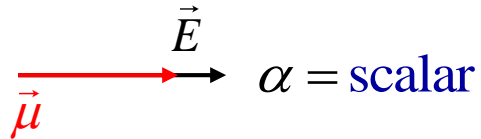
$$\vec{\mu} = \alpha \vec{E} \quad \vec{\mu} = \vec{f}(\vec{E})$$

# Accounting for polarisation in molecular simulation

## Distinguish:

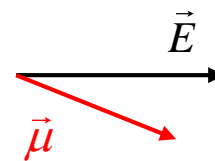
### 4. **Isotropic** versus **anisotropic** polarisation

$$\vec{\mu} = \alpha \vec{E}$$



$\vec{\mu}$  in direction of field  $\vec{E}$

$$\vec{\mu} = \vec{\alpha} \vec{E}$$



$\vec{\alpha}$  is a **tensor**

### 5. **Permanent** dipoles versus **induced** dipoles gas phase                                      due to surroundings in the condensed phase

# Various approximations used

1. **linear dipole** polarisation in a **non-homogeneous electric field**

$$\vec{\mu}(\vec{E}) = \vec{\alpha}\vec{E} + \underbrace{\frac{1}{3}\vec{\alpha}'\vec{\nabla}\vec{E} + \frac{1}{15}\vec{\alpha}''\vec{\nabla}\vec{\nabla}\vec{E}}_{\text{non-uniform part, neglected in homogeneous field approximation}} + \dots$$

2. **linear dipole** polarisation in a **homogeneous electric field**

$$\vec{\mu}(\vec{E}) = \vec{\alpha}\vec{E}$$

3. **non-linear dipole** polarisation in a **homogeneous electric field**

$$\vec{\mu}(\vec{E}) = (\vec{\alpha} + \underbrace{\vec{\beta}\vec{E} + \frac{1}{2}\vec{\gamma}\vec{E}\vec{E} + \dots}_{\text{non-linear part, neglected in linear response approximation}}) \vec{E}$$

# Various approximations used

## 4. isotropic linear dipole polarisation in a homogeneous field

$$\begin{pmatrix} \mu_x \\ \mu_y \\ \mu_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

tensor character, neglected in isotropic polarization approximation

$$\begin{pmatrix} \mu_x \\ \mu_y \\ \mu_z \end{pmatrix} = \alpha \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

scalar

## 5. no (explicit) induced dipoles

$$\vec{\mu}^{\text{permanent}} \neq \vec{0}$$

↑  
may include mean polarisation

$$\vec{\mu}^{\text{induced}}(\vec{E}) = \vec{0}$$

neglect of induced polarisation

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**

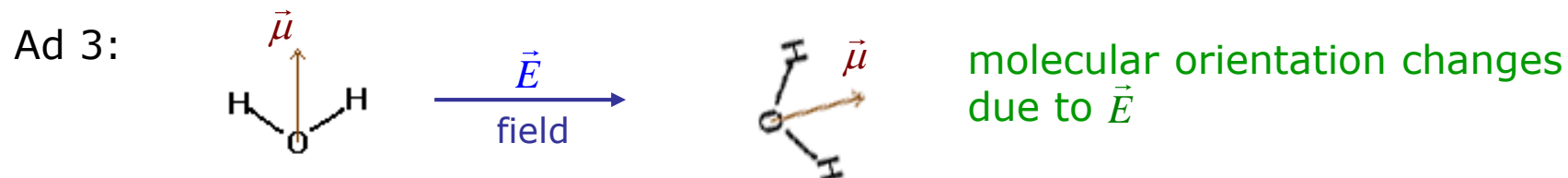
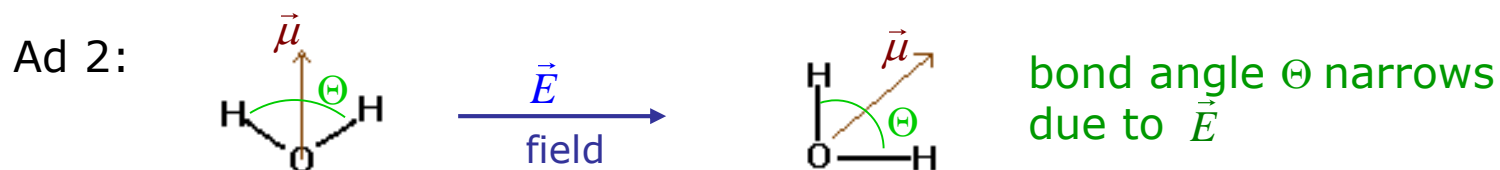
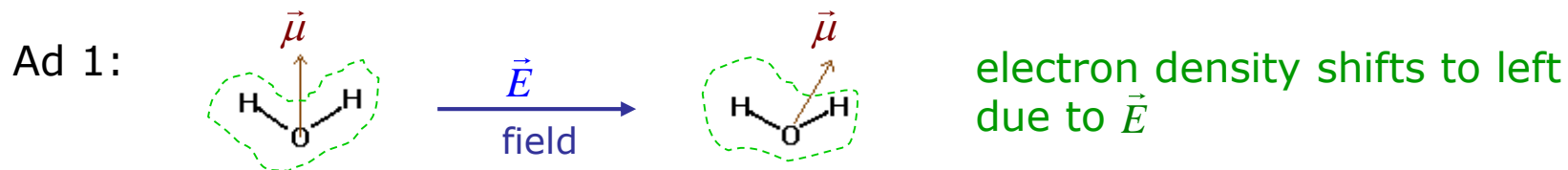
## **C. Charge-on-spring (COS) models**

## **D. Representation of entropy**



# Molecular mechanisms of polarisation

1. **Electronic:** *redistribution of electrons* over atoms/molecules
2. **Geometric:** *change of geometry* of a molecule
3. **Orientational:** *realignment* of molecule



# Inclusion of polarisability in molecular models

## A. Why polarisability ?

1. Effects of polarisability
2. Limitations of non-polarisable force fields

## B. Six aspects

1. Definition, distinctions, approximations
2. Mechanisms
3. Three types of treatments

## C. Charge-on-spring (COS) models

## D. Representation of entropy

# Modelling of polarisability in molecular simulation

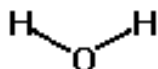
## 1. Mean-field models:

### implicit polarisation

**permanent dipole moment** of a molecule is **enhanced** from gas phase value to the condensed phase value

**for example:**

water



$$\mu_{gas} = 1.85 \text{ Debye}$$

$$\mu_{SPC} = 2.27 \text{ Debye}$$

SPC (simple point charge) model for **liquid** water

**good model for homogeneous environment,  
but not for inhomogeneous surroundings**

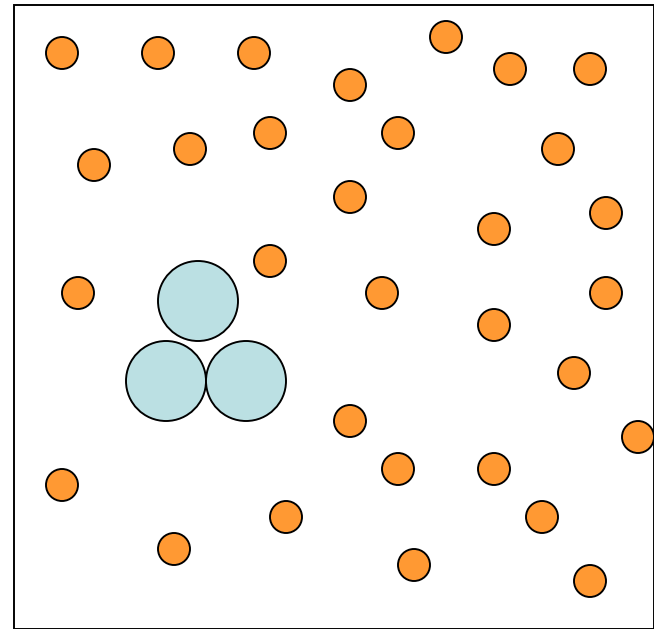
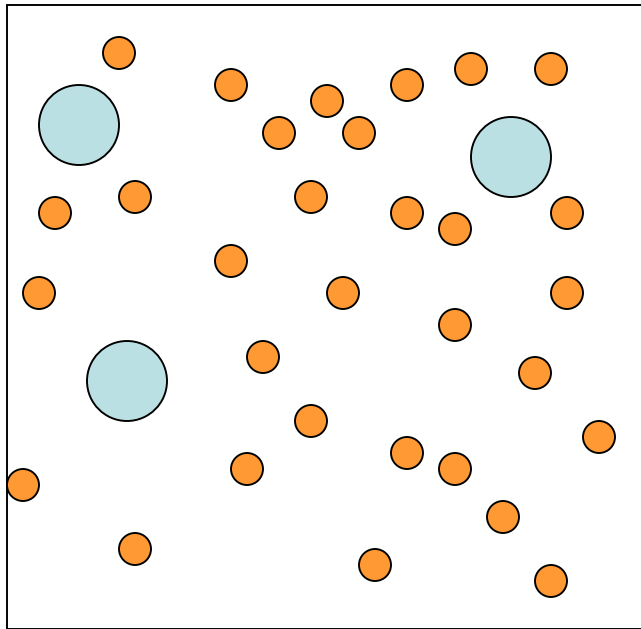
→ H<sub>2</sub>O bound to protein or ion or in a membrane

# The hydrophobic effect

What keeps proteins folded?

Why does oil separate from water?

Why do people who do not know anyone at parties, end up together ?



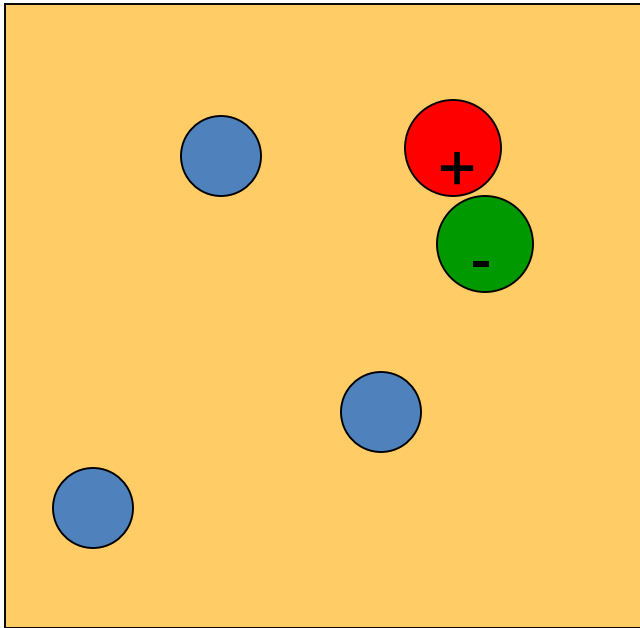
Particles (**blue ones**) are driven together by favourable interactions **within** the environment (**yellow ones**)

***This interaction driving the blue particles together cannot be coarse-grained***

# Modelling or simulating the hydrophobic effect

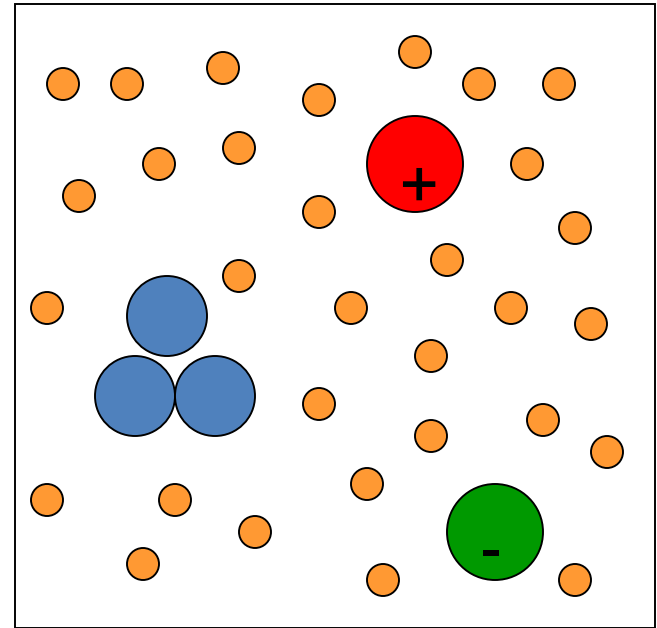
## Implicit solvent:

- *no aggregation of "hydrophobic" solutes*
- *too strong electrostatic interaction: aggregation of unlike charges*
- *no entropy of solvent*



## Explicit solvent:

- *aggregation of "hydrophobic" solutes*
- *damped electrostatic interaction: solvation of charges*
- *entropy of solvent*



It is *NOT* true that hydrophobic particles do not "like" water, rather the interaction of water with water is stronger.  
Ions with unlike charges "like" water more than themselves.

***Implicit solvation models are missing some fundamental biomolecular interactions, which are automatically included in explicit solvent simulations***

# Modelling of polarisability in molecular simulation

## 2. Explicit, classical polarisation in the molecular model

Introduction of *inducible dipoles* in molecules

- **more expensive**
- **not easy to parameterise**

## 3. Quantum-mechanical treatment of electronic degrees of freedom

The electron density can be polarised

- **not efficient for condensed phase systems**  
e.g. ionic solutions

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**

## **C. Charge-on-spring (COS) models**

## **D. Representation of entropy**

# How to introduce classical, explicit polarisability into molecular models for simulation

## A. Generating induced dipoles: 3 methods

1. **induced point dipoles**  $\vec{\mu}_i$  at sites  $\vec{r}_i$

$$\vec{\mu}_i = \alpha_i \vec{E}(\vec{r}_i)$$

2. **changing the size of the (atomic) charges**

$$q_j \text{ at } \vec{r}_j \text{ such that } \vec{\mu}_i = \sum_j \Delta q_j \vec{r}_j = \alpha_i \vec{E}(\vec{r}_i)$$

so-called **fluctuation-charge models**

3. **changing the geometry of the charge distribution**, e.g. by changing the position of virtual charge  $q_j^v$  such that

$$\vec{\mu}_i = \sum_j q_j^v \vec{r}_j^v = \alpha_i \vec{E}(\vec{r}_i)$$

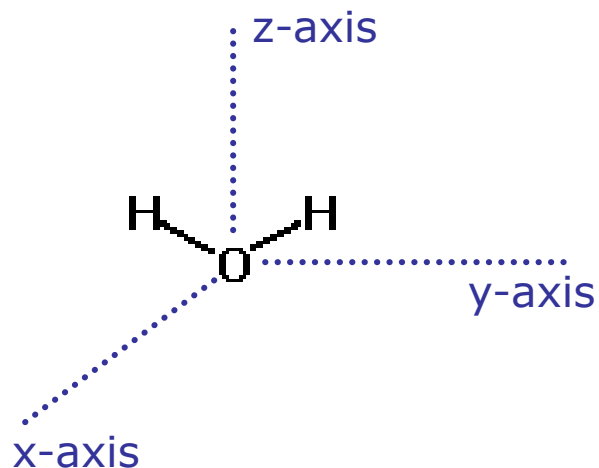
for example: **charge-on-spring model (COS)**



# How to introduce classical, explicit polarisability into molecular models for simulation

## B. Choice of isotropic or anisotropic polarisation

$$\begin{pmatrix} \mu_x \\ \mu_y \\ \mu_z \end{pmatrix} = \begin{pmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \alpha_{yz} \\ \alpha_{zx} & \alpha_{zy} & \alpha_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$



### Water

$$\left. \begin{aligned} \alpha_{xx} &= 1.415 (4\pi\epsilon_0)10^{-3}\text{nm}^3 \\ \alpha_{yy} &= 1.528 (4\pi\epsilon_0)10^{-3}\text{nm}^3 \\ \alpha_{zz} &= 1.468 (4\pi\epsilon_0)10^{-3}\text{nm}^3 \end{aligned} \right\} \alpha = 1.47 (4\pi\epsilon_0)10^{-3}\text{nm}^3$$

off-diagonal elements  
experimentally not known

**bulk of polarisation effect is captured using *isotropic* polarisation**

# How to introduce classical, explicit polarisability into molecular models for simulation

## C. Choosing the sites of the induced dipoles

- **on atoms of model**
- **on other sites:**
  - on bonds (bond polarisability)
  - on virtual atoms


**this may introduce torques**

## D. Avoid the polarisation catastrophe

If two **inducible dipoles** come spatially **too close to each other**, the dipolar interaction between them mutually **enhances their *induced dipoles to infinity***

for example: water  $r_{\text{OO}} \geq \left[ 4\alpha^2 / (4\pi\epsilon_0)^2 \right]^{\frac{1}{6}} \approx 0.144 \text{ nm}$

### **Remedies:**

- damp polarisability at short distances
- **use non-linear polarisation, e.g.**  $\mu = \alpha \ln(E+1)$  *sublinear*  

- use smaller, more distributed polarisabilities

# Considerations regarding choice of model to represent induced dipoles

## Accuracy versus simplicity

### 1. no induced multipoles beyond dipoles

*forces are already 3-rd rank tensors using point dipoles*

### 2. monopoles (charges) are most easy to handle in schemes to compute long-range electrostatic forces

*particle-mesh schemes, virial*

*so: fluctuation-charge or charge-on-spring models*

### 3. rather no rigid virtual sites no anisotropy

} may result in torques

### 4. fluctuation charge models

*scaling the charges of a charge distribution  
does change all multipoles in a fixed ratio*

### 5. charge-on-spring models

*if  $q_j^v$  is large, separation  $\vec{r}_j^v$  is small*

*dipole approximation is good, so, other multipoles don't change*

**The charge-on-spring models are the *simplest* realisations of polarisation**

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**

## **C. Charge-on-spring (COS) models**

## **D. Representation of entropy**

# Induced point dipoles: expression for the energy

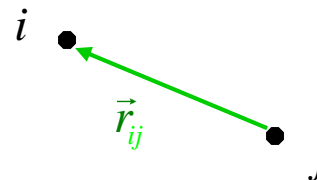
## Induced dipoles:

at sites of  $i=1,2 \dots N$  atoms with permanent charges  $q_i$

$$\vec{\mu}_i = \alpha_i \vec{E}_i = \alpha_i \left\{ \vec{E}_i^q + \sum_{\substack{j=1 \\ j \neq i}}^N \vec{T}_{ij} \vec{\mu}_j \right\} \quad \underline{M} = \underline{A} \left[ \underline{E}^q + \underline{T} \underline{M} \right]$$

## Electrostatic field from permanent charges:

$$\vec{E}_i^q = \sum_{\substack{j=1 \\ j \neq i}}^N \frac{1}{4\pi\epsilon_0} \frac{q_j \vec{r}_{ij}}{r_{ij}^3}$$



## Dipole tensor:

$$\vec{T}_{ij} = \frac{1}{4\pi\epsilon_0} \frac{1}{r_{ij}^3} \left[ \frac{3\vec{r}_{ij} \vec{r}_{ij}}{r_{ij}^2} - 1 \right]$$

# Induced point dipoles: expression for the energy

## Electrostatic energy:

$$U_{el} = U_{qq} + U_{ind} = U_{qq} + U_{stat} + U_{\mu\mu} + U_{self}$$

$$U_{qq} = \frac{1}{2} \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}}$$

$$U_{stat} = - \sum_{i=1}^N \vec{\mu}_i \cdot \vec{E}_i^q$$

$$U_{\mu\mu} = - \frac{1}{2} \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \vec{\mu}_i \vec{T}_{ij} \vec{\mu}_j$$

$$U_{self} = \sum_{i=1}^N \int_0^{\vec{\mu}_i} \vec{E}_i \cdot d\vec{\mu}_i' = \sum_{i=1}^N \int_0^{\vec{\mu}_i} \frac{\vec{\mu}_i'}{\alpha_i} d\vec{\mu}_i' = \sum_{i=1}^N \frac{\vec{\mu}_i^2}{2\alpha_i}$$

# Induced point dipoles: expression for the forces

**At equilibrium:**

$$\frac{\partial U_{el}}{\partial \vec{\mu}_i} = 0 \quad \text{or} \quad \vec{\mu}_i = \alpha_i \vec{E}_i$$

**Force is the negative gradient of  $U_{el}$ :**

$$\begin{aligned} \vec{f}_k &= -\frac{\partial U_{el}}{\partial \vec{r}_k} - \sum_{i=1}^N \frac{\partial U_{el}}{\partial \vec{\mu}_i} \cdot \frac{\partial \vec{\mu}_i}{\partial \vec{r}_k} & \frac{\partial U_{el}}{\partial \vec{\mu}_i} &= 0 \\ &= -\frac{\partial}{\partial \vec{r}_k} \left[ \underbrace{U_{qq} + U_{stat} + U_{\mu\mu}}_{\text{only these contribute}} + U_{self} \right] \end{aligned}$$

# Induced point dipoles: expression for the forces

Force is the negative gradient of  $U_{el}$ :

$$\begin{aligned}\vec{f}_k &= q_k \vec{E}_k^q + \sum_{i=1}^N \vec{\mu}_i \cdot \frac{\partial \vec{E}_i^q}{\partial \vec{r}_k} + \sum_{\substack{i=1 \\ i \neq k}}^N \frac{\partial}{\partial \vec{r}_k} \left( \vec{\mu}_i \vec{T}_{ik} \vec{\mu}_k \right) \\ &= q_k \vec{E}_k^q + \sum_{i=1}^N \vec{\mu}_i \cdot \frac{\partial}{\partial \vec{r}_k} \left( \sum_{\substack{j=1 \\ j \neq k}}^N \frac{1}{4\pi\epsilon_0} \frac{q_j \vec{r}_{ij}}{r_{ij}^3} \right) \\ &\quad + \sum_{\substack{i=1 \\ i \neq k}}^N \frac{1}{4\pi\epsilon_0} \frac{3}{r_{ik}^5} \left\{ \frac{5}{r_{ik}^2} (\vec{\mu}_i \cdot \vec{r}_{ik})(\vec{\mu}_k \cdot \vec{r}_{ik}) \vec{r}_{ik} - (\vec{\mu}_i \cdot \vec{\mu}_k) \vec{r}_{ik} - (\vec{\mu}_i \cdot \vec{r}_{ik}) \vec{\mu}_k - (\vec{\mu}_k \cdot \vec{r}_{ik}) \vec{\mu}_i \right\}\end{aligned}$$

if the induced dipoles  $\vec{\mu}_i$  are known, force  $\vec{f}_k$  is known



# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**
- 6. Techniques to solve the field equations**

## **C. Charge-on-spring (COS) models**

## **D. Representation of entropy**

# Equation for the induced dipoles or field equation

**Induced dipoles:**  $\vec{\mu}_i = \alpha_i \vec{E}_i = \alpha_i \left\{ \vec{E}_i^q + \sum_{\substack{j=1 \\ j \neq i}}^N \vec{T}_{ij} \vec{\mu}_j \right\}$

**Induced dipoles equation:**  $\vec{\mu}_i = \alpha_i \left\{ \vec{E}_i^q + \sum_{\substack{j=1 \\ j \neq i}}^N \vec{T}_{ij} \vec{\mu}_j \right\}$

**Field equation:**  $\vec{E}_i = \left\{ \vec{E}_i^q + \sum_{\substack{j=1 \\ j \neq i}}^N \alpha_j \vec{T}_{ji} \vec{E}_j \right\}$

# Equation for the induced dipoles or field equation

Using matrix notation:

$$\underline{\underline{M}} = \begin{pmatrix} \vec{\mu}_1 \\ \vdots \\ \vec{\mu}_N \end{pmatrix} \quad \underline{\underline{A}} = \begin{pmatrix} \alpha_1 & & \\ & \alpha_2 & \\ & & \ddots \\ & & & \alpha_N \end{pmatrix} \quad \underline{\underline{T}} = \text{dipole tensor}$$

$$\underline{\underline{T}}_{ij} = \frac{1}{4\pi\epsilon_0} \frac{1}{r_{ij}^3} \left\{ \frac{3}{r_{ij}^2} \begin{pmatrix} x_{ij}x_{ij} & x_{ij}y_{ij} & x_{ij}z_{ij} \\ y_{ij}x_{ij} & y_{ij}y_{ij} & y_{ij}z_{ij} \\ z_{ij}x_{ij} & z_{ij}y_{ij} & z_{ij}z_{ij} \end{pmatrix} - \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \right\}$$

Induced dipole equation:

$$\underline{\underline{M}} = \underline{\underline{A}}\underline{\underline{E}} = \underline{\underline{A}}\left[\underline{\underline{E}}^q + \underline{\underline{T}}\underline{\underline{M}}\right]$$

or

$$\left[\underline{\underline{A}}^{-1} - \underline{\underline{T}}\right]\underline{\underline{M}} = \underline{\underline{E}}^q \quad \text{so} \quad \underline{\underline{M}} = \left[\underline{\underline{A}}^{-1} - \underline{\underline{T}}\right]^{-1} \underline{\underline{E}}^q$$

# Methods to solve the field equations or to find the induced dipoles

## Four methods

**1. matrix inversion:** invert  $\left[ \underline{A}^{-1} - \underline{T} \right]$   
 $3N \times 3N$  matrix  $\rightarrow$  effort proportional to  $N^3$  **expensive**

**2. iterative solution:**

$$\underline{M}^{(n)} = \underline{A} \left[ \underline{E}^q + \underline{T} \underline{M}^{(n-1)} \right] \quad n = 1, 2, \dots$$

**3. use a prediction from previous MD steps (plus iteration)**

$$\vec{\mu}_i(t) = 2\vec{\mu}_i(t - \Delta t) - \vec{\mu}_i(t - 2\Delta t)$$

**efficient method (combined with iteration)**

# Methods to solve field equations or to find the induced dipoles

## Four methods

### 4. Treat $\vec{\mu}_i$ as additional dynamical variables

*extended system technique:*

*Lagrangian such that  $\langle \vec{\mu}_i \rangle = \alpha_i \langle \vec{E}_i \rangle$*

$$L = \frac{1}{2} \sum_{i=1}^N m \dot{\vec{r}}_i^2 + \frac{1}{2} \sum_{i=1}^N M_i^\mu \dot{\vec{\mu}}_i^2 - \frac{1}{2} \sum_{i=1}^N \sum_{\substack{j=1 \\ j \neq i}}^N \frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}} \\ - \sum_{i=1}^N \vec{\mu}_i \cdot \vec{E}_i + \sum_{i=1}^N \frac{\vec{\mu}_i^2}{2\alpha_i}$$

**Small time step required, fluctuations induce dispersion forces**

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**
- 6. Techniques to solve the field equations**

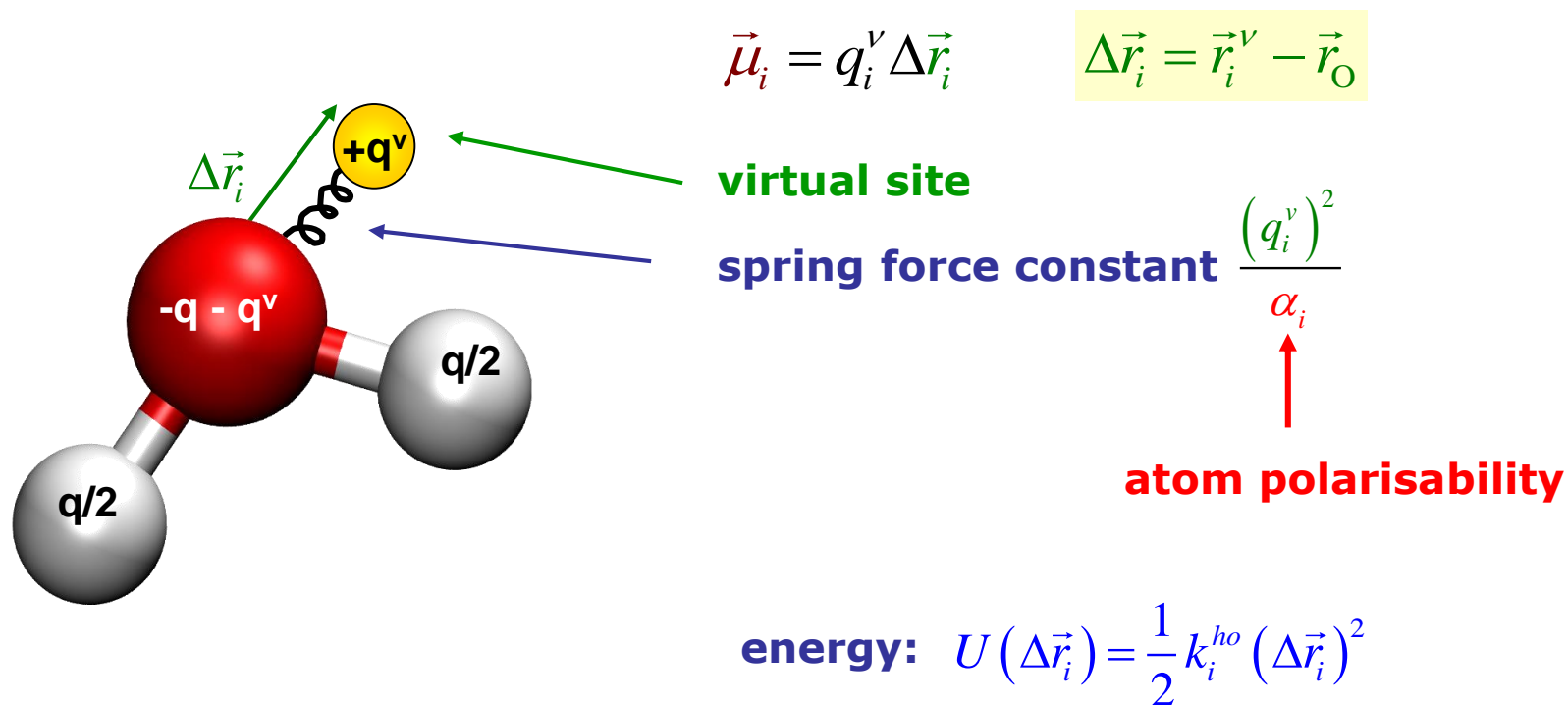
## **C. Charge-on-spring (COS) models**

- 1. Equations, parameters, non-linear polarisation**

## **D. Representation of entropy**

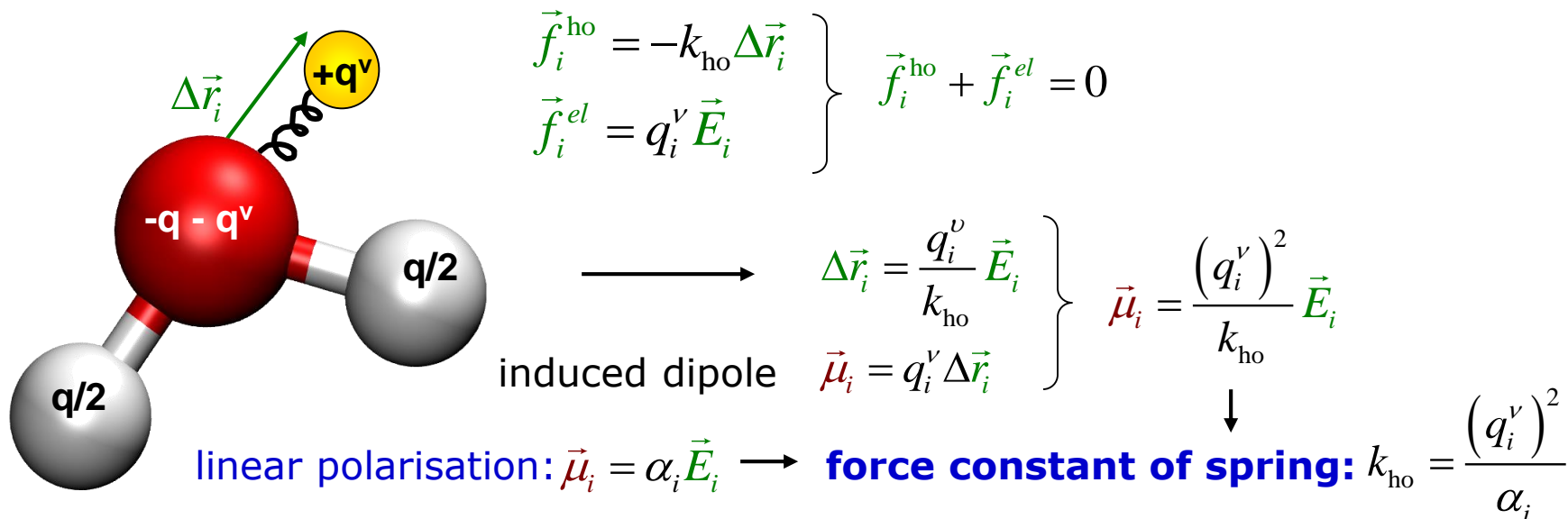
# The Charge-On-Spring (COS) model

The **simplest realisation of polarisation** is the **displacement** of a **charge**  $q^v$  at a **virtual (massless) site** which is **restrained by** a harmonic potential (a **spring**) to one of the real (atomic) sites or to a virtual site. Of course there is no intra-molecular Coulomb interaction.



# The Charge-On-Spring Model

The **virtual particle i** with charge  $q^v$  must at any time be positioned such that the **total force acting on it is zero**



## Parameters of the model:

1. **site**  $i$  of charge on spring
2.  $q_i^v$  if chosen large ( $-8e$ ) then  $\Delta \vec{r}_i$  small
3.  $\alpha_i$  polarisability



# Problems with polarisable force fields

Including polarisation in a force field can lead to overpolarisation with effects such as polarisation catastrophe and too large dielectric permittivity

## Some remedies:

A	large enough repulsive van der Waals term	all COS models
B	reduced polarisability	COS/B2, COS/G2
C	virtual COS site	COS/G2, <b>COS/D2</b>
D	sublinear dependence on electric field	<b>COS/D2</b>
E	several COS sites	expensive
F	screening for short distances	ad hoc

*Yu et al., J. Chem. Phys 118 (2003) 221-234*

*Yu & van Gunsteren, J. Chem. Phys 121 (2004) 9549-9564*

*Kunz & van Gunsteren, J. Phys. Chem. A113 (2009) 11570-11579*

*Bachmann & van Gunsteren, Mol. Phys. 112 (2014) 2761-2780*

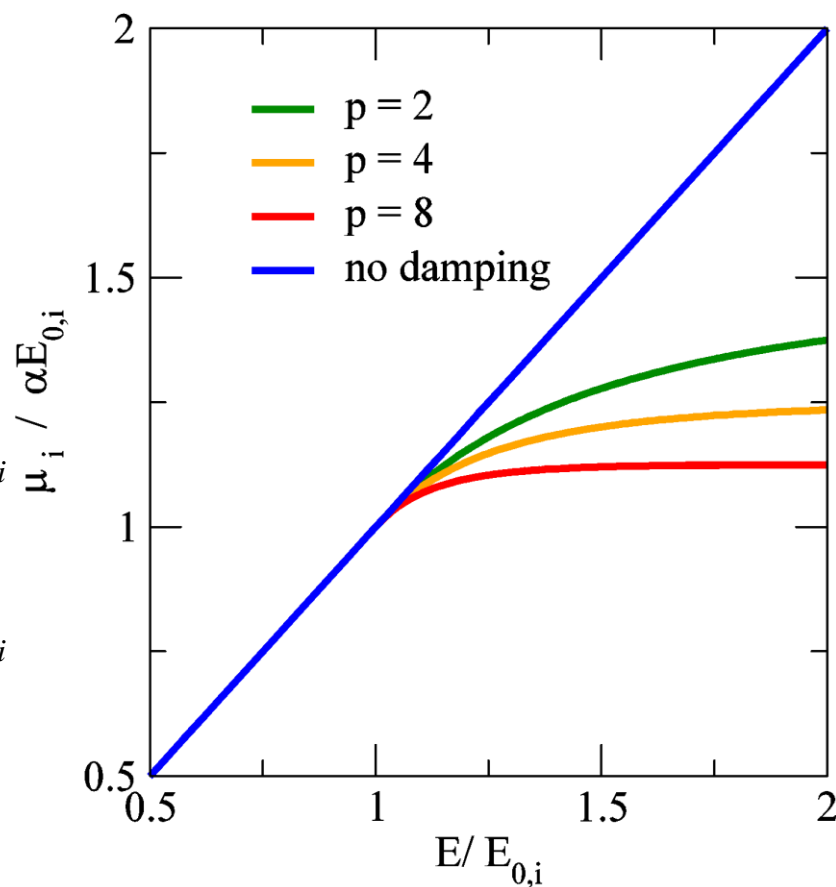
*Bachmann & van Gunsteren, J. Chem. Phys. 141 (2014) 22D515*

# Damped non-linear polarisability

Induced dipole **linear** up to a certain value of the electric field **then levelling off** (saturation).

**Continuous** and continuous in the **first derivative**:

$$\vec{\mu}_i = \begin{cases} \alpha_i \vec{E}_i & |\vec{E}_i| < E_{0,i} \\ \frac{\alpha_i E_{0,i}}{p} \left[ p + 1 - \left( \frac{E_{0,i}}{|\vec{E}_i|} \right)^p \right] \hat{E}_i & |\vec{E}_i| \geq E_{0,i} \end{cases}$$



**Accounts for hyperpolarisability in a simplified way.**

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**
- 6. Techniques to solve the field equations**

## **C. Charge-on-spring (COS) models**

- 1. Equations, parameters, non-linear polarisation**
- 2. Applications: liquid water and other solvents**

## **D. Representation of entropy**

# An example of a *sub-atomic* water model: The COS/D2 polarisable model

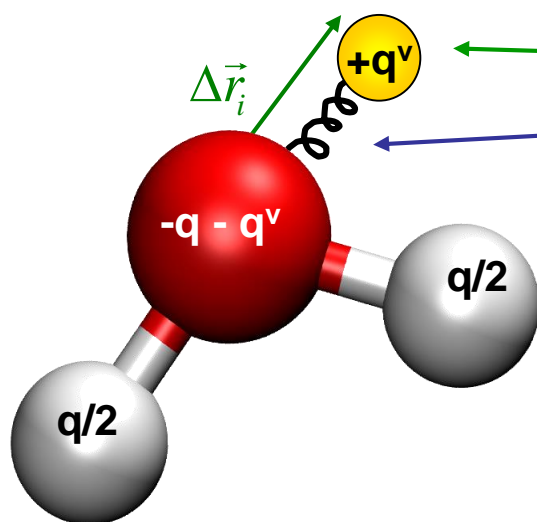
- Four-site (FG) model

The **simplest realisation of polarisation** is the **displacement** of a **charge**  $q^v$  at a **virtual (massless) site** which is **restrained by** a harmonic potential (a **spring**) to one of the real (atomic) sites or to a virtual site.

Inducible electric dipole

$$\vec{\mu}_i = q_i^v \Delta \vec{r}_i$$

$$\Delta \vec{r}_i = \vec{r}_i^v - \vec{r}_o$$



virtual site  
spring force constant  $\frac{(q_i^v)^2}{\alpha_i}$

$\alpha_i$

atom polarisability

energy:  $U(\Delta \vec{r}_i) = \frac{1}{2} k_i^{ho} (\Delta \vec{r}_i)^2$

# Force-field parameters: water

non-polarisable    polarisable

	Exp.	SPC	COS/G2	COS/D2
$C_6$ (O-O) ( $10^{-3}$ kJmol $^{-1}$ nm $^6$ )		2.617	3.244	3.2444
$C_{12}$ (O-O) ( $10^{-6}$ kJmol $^{-1}$ nm $^{12}$ )		2.634	3.458	3.252
$C_{12}$ (H-H) ( $10^{-6}$ kJmol $^{-1}$ nm $^{12}$ )				3.0
$\mu_0$ (D)	<b>1.855</b>	<b>2.270</b>	1.850	<b>1.855</b>
$\alpha$ ( $4\pi\epsilon_0$ $10^{-3}$ nm $^3$ )	<b>1.494</b>		<b>1.255</b>	<b>1.3</b>

$\mu_0$   
increased

$\alpha$   
reduced

$\alpha$   
reduced

- Geometry:  $d_{OH}$  : 0.09572 nm     $\angle HOH$ : 104.52°
- Virtual site M:  $d_{OM}$  : 0.02472 nm    (COS/G2 : 0.022 nm)
- Charge on spring: -8e
- Damping:  $p$  : 8     $E_0$  : 140 (kJmol $^{-1}$ nm $^{-3}$ ) $^{1/2}$

# Results: thermodynamic quantities, water

non-polarisable polarisable

		Exp	SPC	COS/G2	COS/D2
$\rho$	(kgm <sup>-3</sup> ) density	997	973	999	999
$\Delta H_{vap}$	(kJmol <sup>-1</sup> ) heat of vaporisation	44.1	43.9	43.7	44.1
$\Delta F_{exc}$	(kJmol <sup>-1</sup> ) excess free energy	24.0	23.6	21.8	24.3
$\gamma$	(mNm <sup>-1</sup> ) surface tension	71.6	48.4	59.0	63.6
$C_p$	(Jmol <sup>-1</sup> K <sup>-1</sup> ) heat capacity	75.3	93.0	107.7	88.9
$\alpha_p$	(10 <sup>-4</sup> K <sup>-1</sup> ) thermal expansion	4.22	9.0	7.0	4.9
$\kappa_T$	(10 <sup>-6</sup> atm <sup>-1</sup> ) compressibility	45.8	47.8	47.8	44.4

$\rho$  too small

$\gamma$  too small

$C_p$  too large

$\alpha_p$  too large

$\gamma$  too small

$C_p$  too large

$\alpha_p$  too large

$\gamma$  too small

$C_p$  too large

  parameterised against

# Results: dynamic quantities, water

non-polarisable   polarisable

	Exp	SPC	COS/G2	COS/D2
$D$ ( $10^{-9} \text{ m}^2\text{s}^{-1}$ ) diffusion constant	2.3	4.2	2.0	2.2
$\tau_2^{OH}$ (ps) rotational correlation time	1.95	1.9	3.9	3.8

$D$  too large    $\tau_2^{OH}$  too large    $\tau_2^{OH}$  too large

# Results: electric quantities, water

non-polarisable    polarisable

	Exp.	SPC	COS/G2	<b>COS/D2</b>
$\epsilon(0)$ dielectric permittivity	<b>78.4</b>	<b>64.7</b>	<b>87.8</b>	78.9
$\langle \mu \rangle$ (D)    total molecular dipole	2.3 – 2.5	2.27	2.61	2.55
$\langle \mu^{ind} \rangle$ (D) induced molecular dipole	-	-	0.80	0.73
$\tau_D$ (ps)    Debye relaxation time	<b>8.3</b>	<b>6.9</b>	<b>15.8</b>	8.4

$\epsilon(0)$  too small     $\epsilon(0)$  too large  
 $\tau_D$  too small     $\tau_D$  too large

*Bachmann & van Gunsteren, J. Chem. Phys. 141 (2014) 22D515*



# Properties of non-polarisable and polarisable liquid chloroform $CHCl_3$

Properties	Experiment	Non-polarisable model (G96) 4 sites		Polarisable model (COS/C) 5 sites	
$\rho$ (kgm <sup>-3</sup> ) density	1489		1504		1489
$\Delta H_{vap}$ (kJmol <sup>-1</sup> ) heat of vaporisation	33.4		31.2		33.3
$\Delta F_{exc}$ (kJmol <sup>-1</sup> ) excess free energy	15.3		15.1		16.6
$\gamma$ (10 <sup>-3</sup> Nm <sup>-1</sup> ) surface tension	27.2		25.8		28.7
$C_V$ (Jmol <sup>-1</sup> K <sup>-1</sup> ) heat capacity	74.7		71.7		73.5
$\alpha_p$ (10 <sup>-3</sup> K <sup>-1</sup> ) thermal expansion	1.26		1.44		1.29
$\kappa_T$ (10 <sup>-10</sup> Pa <sup>-1</sup> ) compressibility	9.98		14.2		9.2
$\epsilon(0)$ dielectric permittivity	4.81		2.3		3.7
$\epsilon(\infty)$					1.2
$\tau_D$ (ps) Debye relaxation time	5.4		4.3		5.6
$D$ (10 <sup>-9</sup> m <sup>2</sup> s <sup>-1</sup> ) diffusion	2.3		2.4		2.2
$\eta$ (cP) viscosity	0.56		0.48		0.55

Overall “better” model

☐ parametrised against

*Z. Lin et al., Mol. Phys. 108 (2010) 1749-1757*

*I.G. Tironi & WFvG, Mol Phys. 83 (1994) 381-403*

## Properties of non-polarisable and polarisable liquid $CCl_4$

Properties	Experiment	Non-polarisable model (G96) 4 sites (smaller system)	Polarisable model (COS/C) 5 sites (longer simulation)
$\rho$ (kgm <sup>-3</sup> ) density	1594	1601	1600
$\Delta H_{vap}$ (kJmol <sup>-1</sup> ) heat of vaporisation	32.4	32.4	32.4
$\Delta F_{exc}$ (kJmol <sup>-1</sup> ) excess free energy	16.3	15.8	15.8
$\gamma$ (10 <sup>-3</sup> Nm <sup>-1</sup> ) surface tension	26.8	28.8	28.8
$\epsilon(0)$ dielectric permittivity	2.24	1.0	2.24
$D$ (10 <sup>-9</sup> m <sup>2</sup> s <sup>-1</sup> ) diffusion	1.15	1.8	1.9
$\eta$ (cP) viscosity	0.96	0.88	0.88

Better representation  
of dielectric response

 parametrised against

*A.P.E. Kunz et al., Mol. Phys. 109 (2011) 365-372*  
*I.G. Tironi et al., Mol Sim. 18 (1996) 1-11*

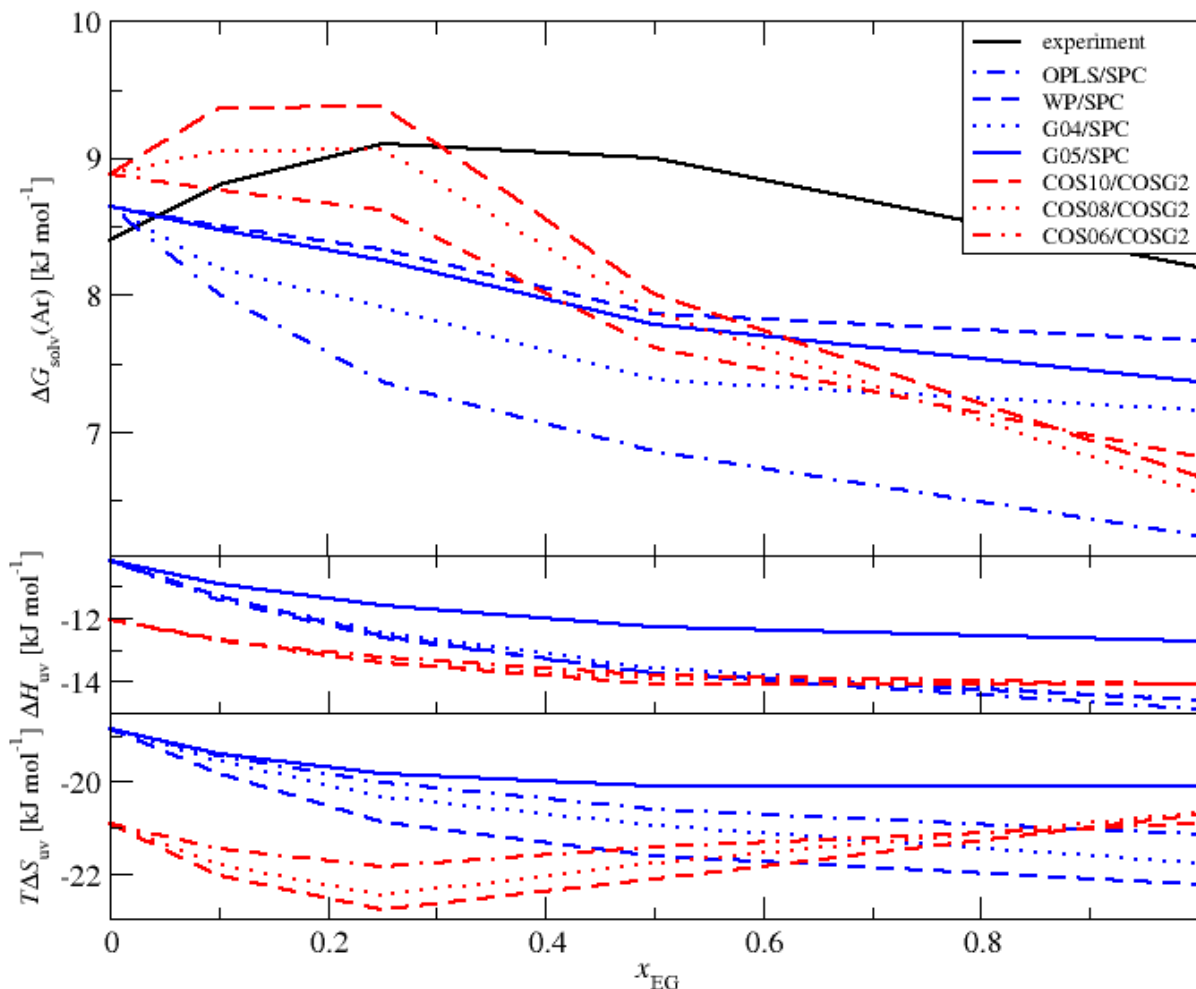
# Hydrophobic hydration (argon) in water-ethylene-glycol mixtures

Non-polarisable models versus polarisable models

Free enthalpy  
of solvation

Solute-solvent  
enthalpy  
of solvation

Solute-solvent  
entropy  
of solvation



Black:  
experiment

Only polarisable models reproduce the experimental trend

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**
- 6. Techniques to solve the field equations**

## **C. Charge-on-spring (COS) models**

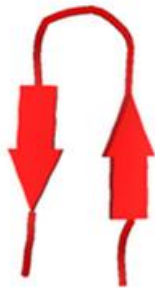
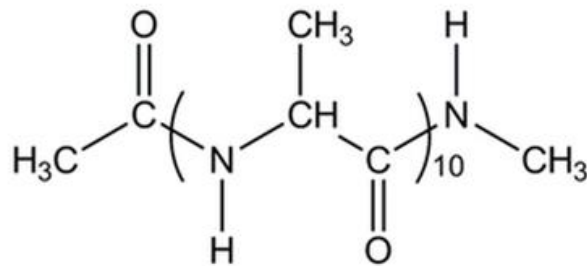
- 1. Equations, parameters, non-linear polarisation**
- 2. Applications: liquid water and other solvents**
- 3. Applications: secondary structure preferences of solutes**

## **D. Representation of entropy**

# Stabilisation of $\alpha$ -helical versus $\beta$ -hairpin secondary structure by polarisability of the solvent

A deca-Ala-peptide in **water**, **methanol**, **chloroform** or  **$\text{CCl}_4$**

Free enthalpy difference between **helix** and **hairpin** upon introduction of polarisability into the **solvent**



Solvent	Free enthalpy difference		Molecular polarisability $\alpha$ [ $(4\pi\epsilon_0)10^{-3} \text{ nm}^3$ ]
	$(G^\alpha_{\text{pol}} - G^\alpha_{\text{nonpol}}) - (G^\beta_{\text{pol}} - G^\beta_{\text{nonpol}})$ [kJ mol $^{-1}$ ]	Per residue [kJmol $^{-1}$ ]	
water	$0.5 \pm 3.7$	0.0	0.93
methanol	$-11.5 \pm 3.5$	-1.1	1.32
chloroform	$-10.2 \pm 1.4$	-1.0	9.5
$\text{CCl}_4$	$-21.0 \pm 0.6$	-2.1	11.1

**Polarisability becomes more important for non-polar solvents or environments, e.g. tails of lipids in membranes**

# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**
- 6. Techniques to solve the field equations**

## **C. Charge-on-spring (COS) models**

- 1. Equations, parameters, non-linear polarisation**
- 2. Applications: liquid water and other solvents**
- 3. Applications: secondary structure preferences of solutes**

## **D. Representation of entropy**

# Energy-entropy compensation in solvents

Atomic-level, fine-grained (FG), (non-)polarisable models

Supra-molecular, coarse-grained (CG), polarisable models

		Free energy	Energy	$T\Delta S_{vap}$ Entropy	$\frac{T\Delta S_{vap}}{\Delta H_{vap}}$
		$\Delta F_{exc}$ $\text{kJ mol}^{-1}$	$\Delta H_{vap}$ $\text{kJ mol}^{-1}$	$\Delta H_{vap} - \Delta F_{exc}$ $\text{kJ mol}^{-1}$	
Water					
	experiment	24.0	44.0	20.0	0.45
	polarisable FG model COS/G2	21.8	43.7	21.9	0.50
	non-polarisable FG model SPC	23.6	43.9	20.3	0.46
	polarisable CG model CGW	11.0	25.9	14.9	0.58
Methanol					
	experiment	17.8	38.1	20.3	0.53
	polarisable FG model COS/M	14.7	37.8	23.1	0.61
	non-polarisable FG model	16.7	37.9	21.2	0.56
	polarisable CG model CGM	7.3	18.0	10.7	0.59

Energy – entropy compensation:  $T\Delta S_{vap} \approx \frac{1}{2} \Delta H_{vap}$

Supra-molecular (CG) and atomic (FG) polarisable or non-polarisable models of water (H<sub>2</sub>O) and methanol (MeOH) must reproduce the entropy/enthalpy ratio of the liquid



# **Inclusion of polarisability in molecular models**

## **A. Why polarisability ?**

- 1. Effects of polarisability**
- 2. Limitations of non-polarisable force fields**

## **B. Six aspects**

- 1. Definition, distinctions, approximations**
- 2. Mechanisms**
- 3. Three types of treatments**
- 4. Three types of models, choices to be made, considerations**
- 5. Expressions for the energy and forces (point dipoles)**
- 6. Techniques to solve the field equations**

## **C. Charge-on-spring (COS) models**

- 1. Equations, parameters, non-linear polarisation**
- 2. Applications: liquid water and other solvents**
- 3. Applications: secondary structure preferences of solutes**

## **D. Representation of entropy**



# Accounting for polarisation effects in bio-molecular simulation

**For bio-molecular solutes: proteins, DNA, RNA, ...**

**use quantum-chemical models,  
classical polarisability models are too crude  
to properly represent the electron redistribution**

**For solvents and lipids in membranes:**

**use classical (Charge-On-Spring) models,  
quantum-chemical models are too expensive or inaccurate**

# Acknowledgements

Gruppe informatikgestützte Chemie (igc)

<http://www.igc.ethz.ch>



*Victor Rusu Holanda* (Brazil)

**Associated groups:**



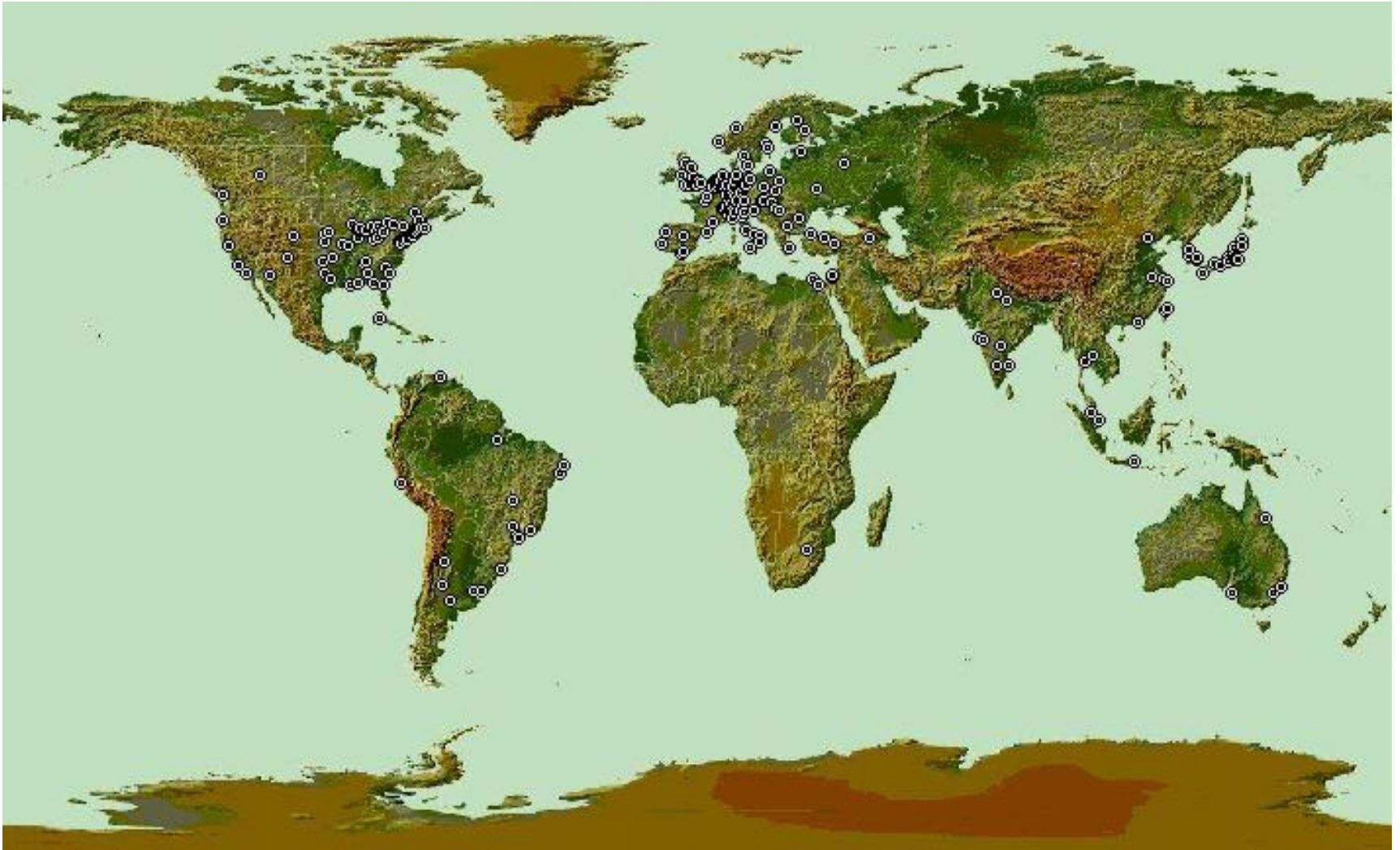
*Philippe Hünenberger  
and co-workers*



*Sereina Riniker  
and co-workers*

# Spatial distribution of licences

## GROMOS biomolecular simulation software



**GROMOS = Groningen Molecular Simulation + GROMOS Force Field**

Generally available: <http://www.gromos.net>