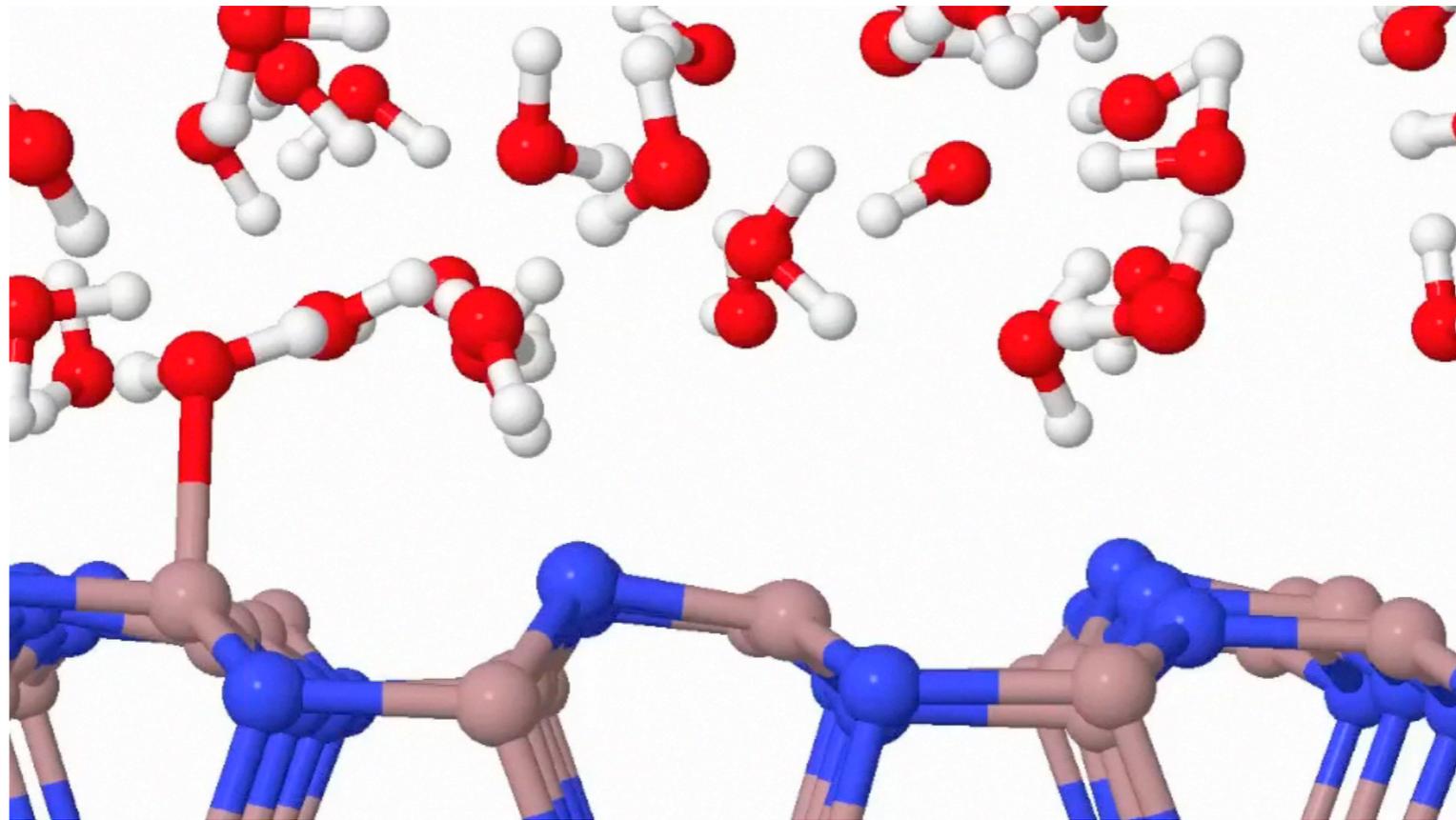


Role of Polarizability in the Structure and Dynamics of liquid water

Marivi Fernandez-Serra
Stony Brook University





ANOMALIES IN WATER



Superco

Isotherm



coefficient

(c)

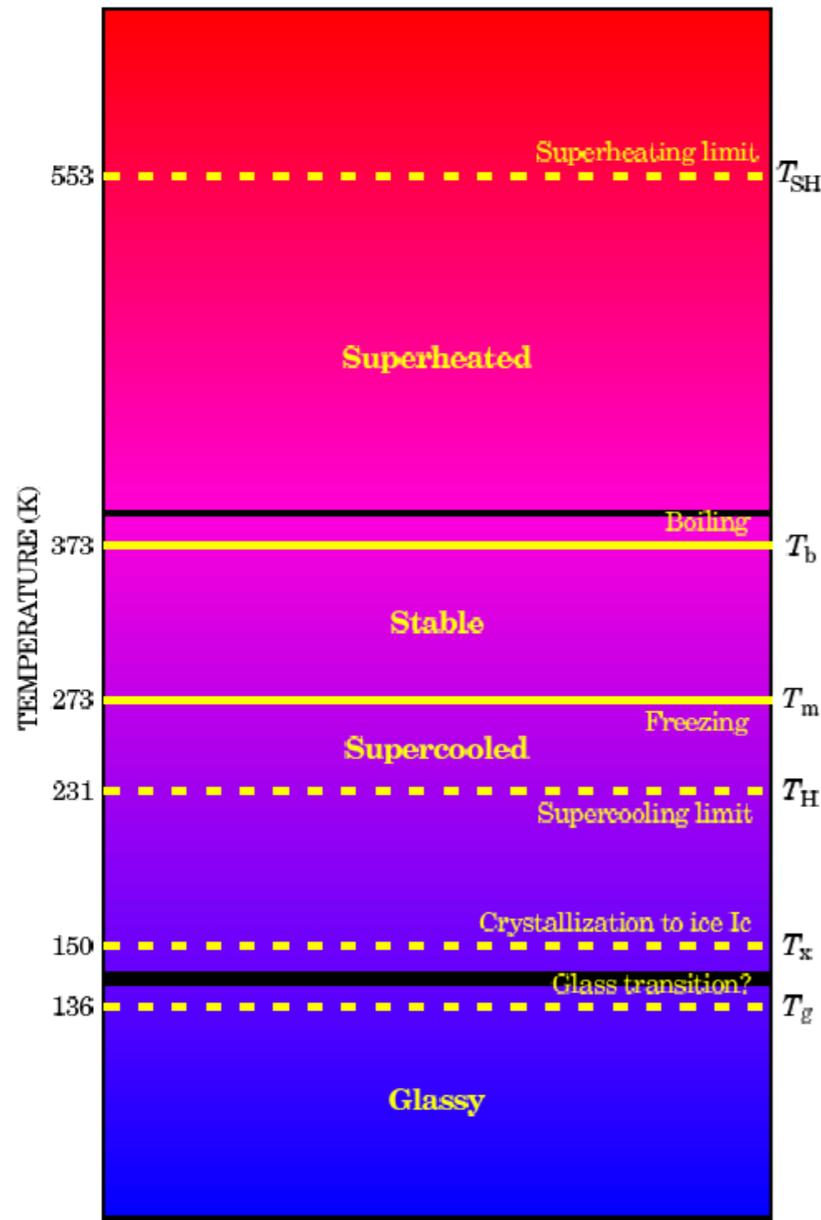
To e
locat

Poole, E

point



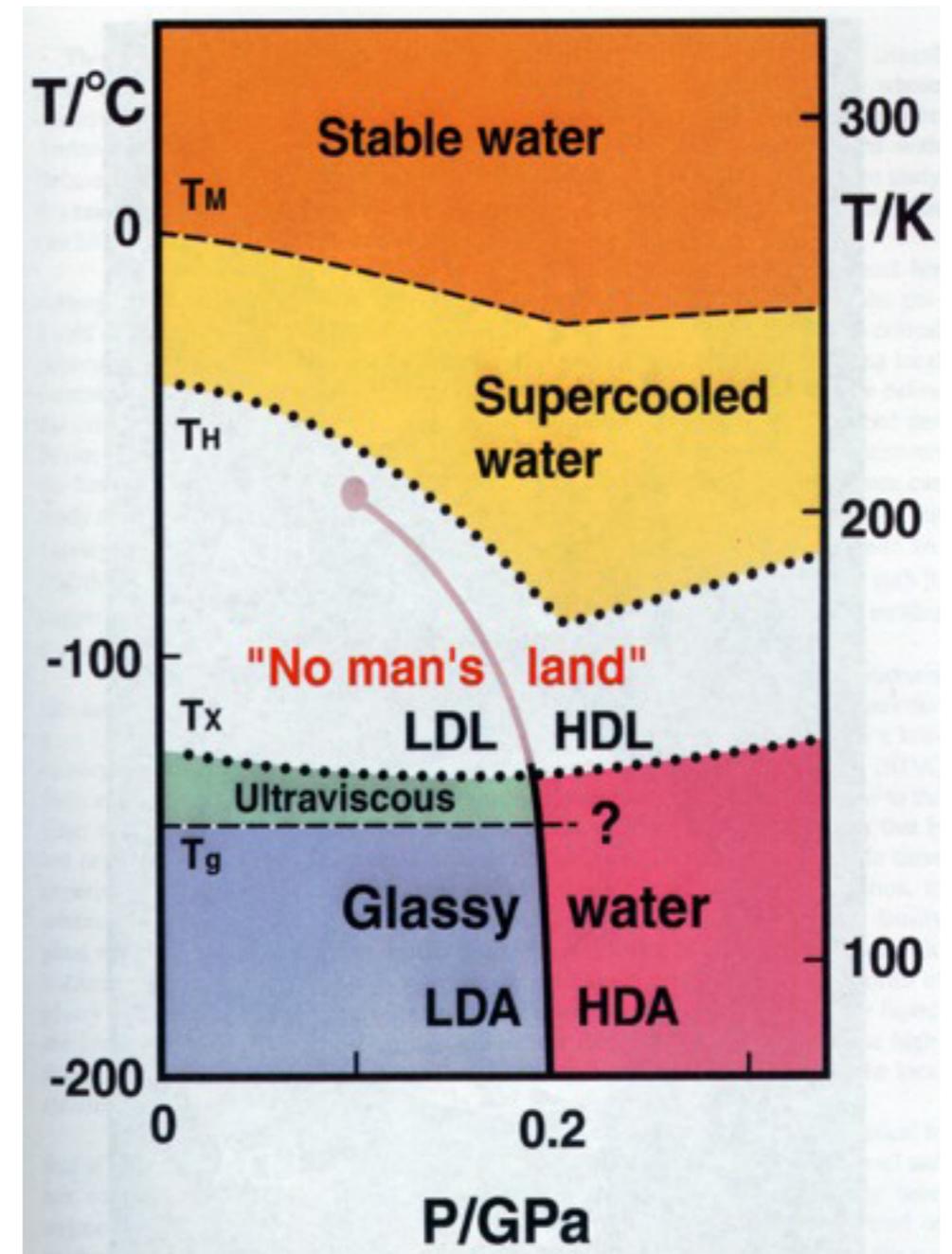
OVERSIMPLIFIED PHASE DIAGRAM



Poole, P. H., Sciortino, F., Essmann, U., Stanley, H. E. (1992) *Nature* 360:324–328

G. Debenedetti and H. E. Stanley, "Supercooled and Glassy Water," *Physics Today* 56 [issue 6], 40-46 (2003)

“hypothesized phase diagram”



Homogeneous nucleation temperature $T_M=231$ K: T cannot be supercooled below this.
 $T_{\text{glass}}=130$ K: LDA melts into a highly viscous liquid.
 $T_x=150$ K, a highly viscous liquid crystallizes into cubic ice Ic
 $150 \text{ K} < \text{No man's Land} < T_m (231 \text{ K})$



TWO FLUCTUATING LIQUIDS



REVIEW

Received 9 Jun 2015 | Accepted 26 Oct 2015 | Published 8 Dec 2015

DOI: 10.1038/ncomms9998

OPEN

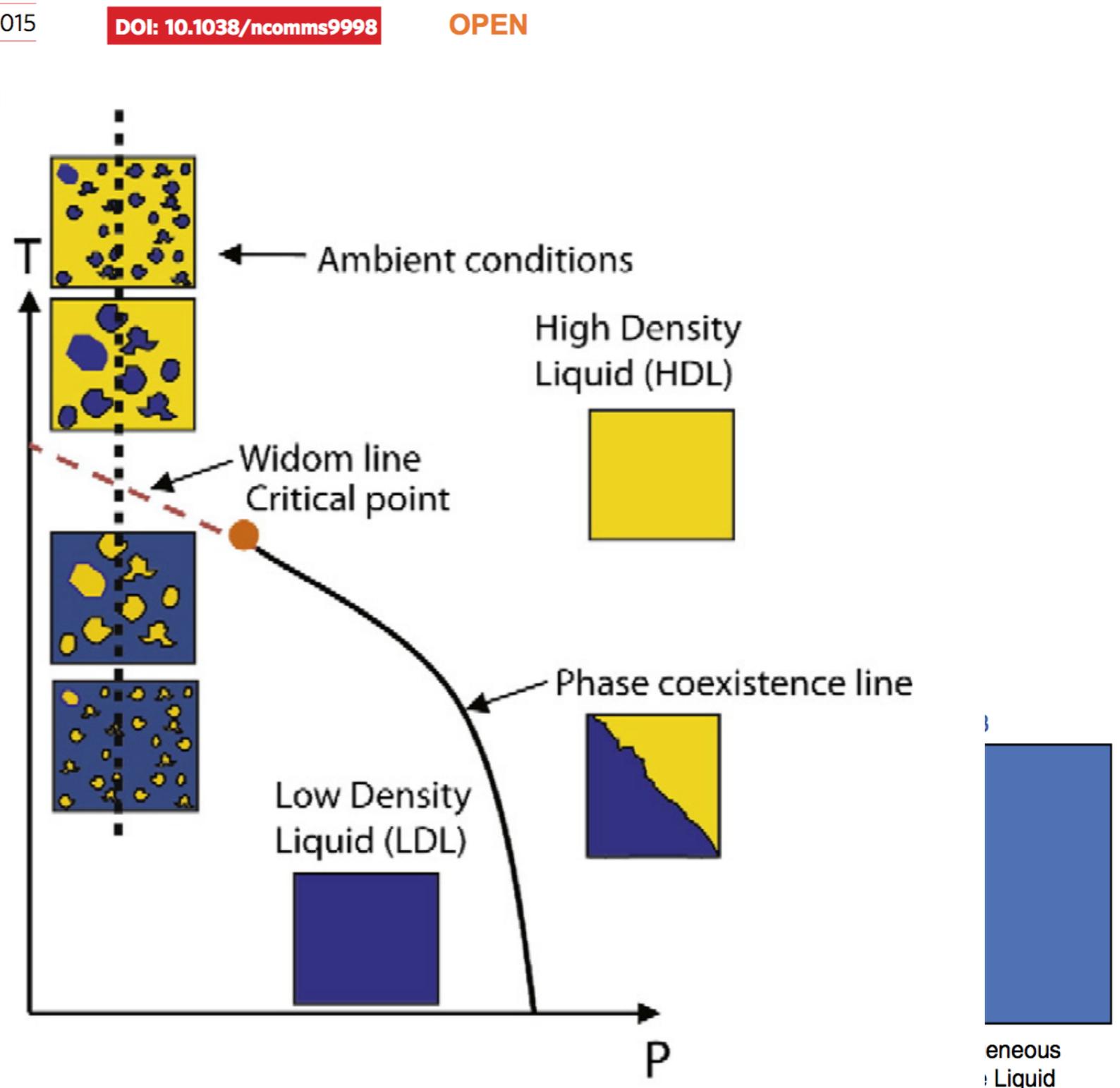
The structural origin of a liquid water

Anders Nilsson¹ & Lars G.M. Pettersson¹

How can the hypothesis be tested?

- Experiments: No access to no ma interpretation, confined water.
- Models: 2-scale models, designed LLCP
- Simulations: Based on fitted mode reproduce the full phase diagram

If experiments are not conclusive, only well converged ab initio simulations can give an unbiased answer





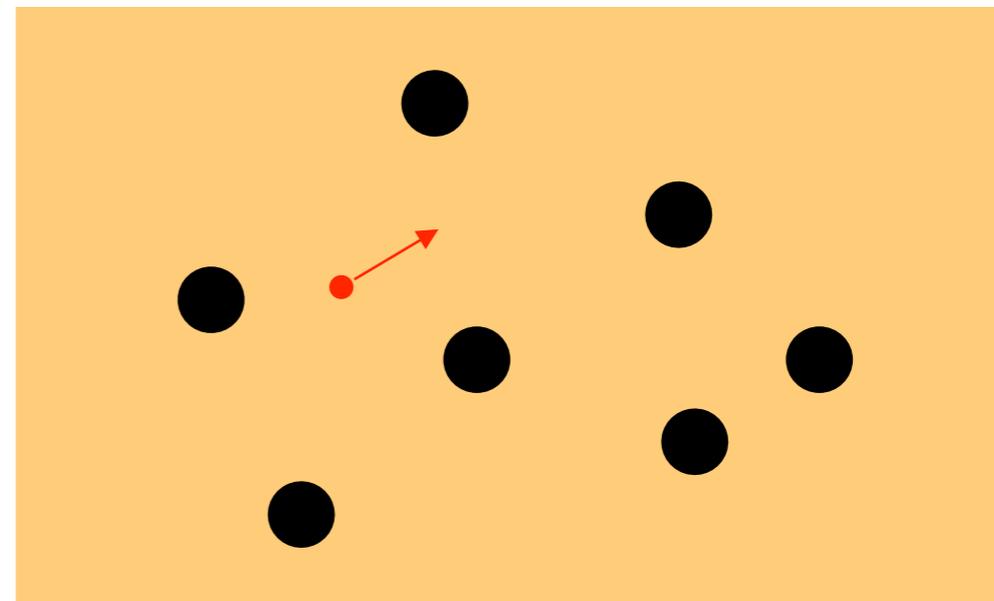
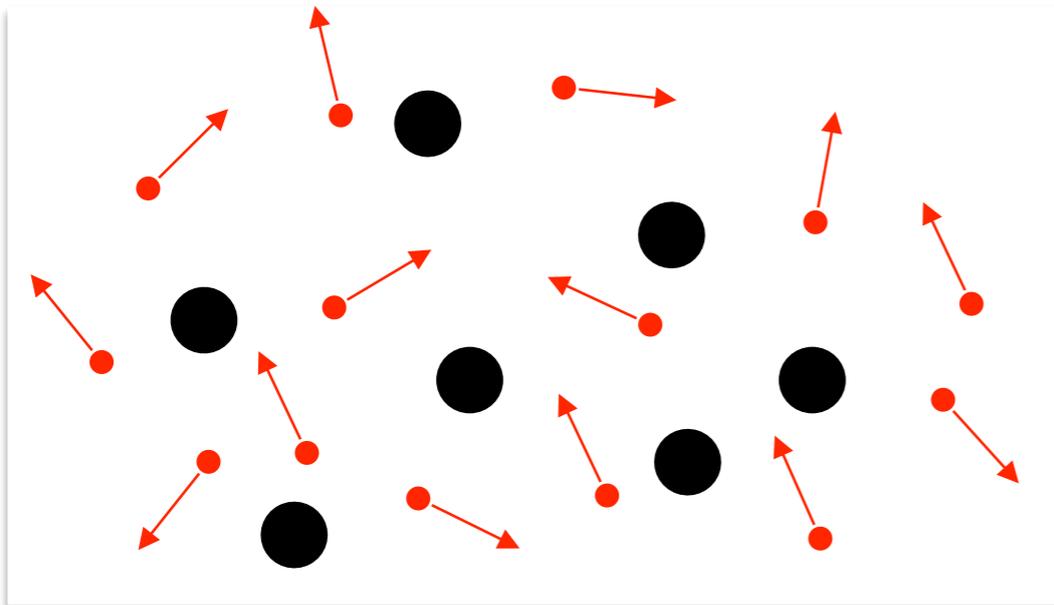
DENSITY FUNCTIONAL THEORY



$$H = \sum_{i=1}^{n_e} \left[-\frac{\hbar}{2m_e} \nabla_i^2 + V_N(\mathbf{r}_i) \right] + \frac{1}{2} \sum_{\substack{i,j=1 \\ i \neq j}}^{n_e} \frac{e^2}{|\mathbf{r}_i - \mathbf{r}_j|}$$

$$H_{DFT} = \sum_{i=1}^{n_e} \left[-\frac{\hbar}{2m_e} \nabla_i^2 + V_N(\mathbf{r}_i) + V_H(\mathbf{r}_i) + V_{XC}(\mathbf{r}_i) \right]$$

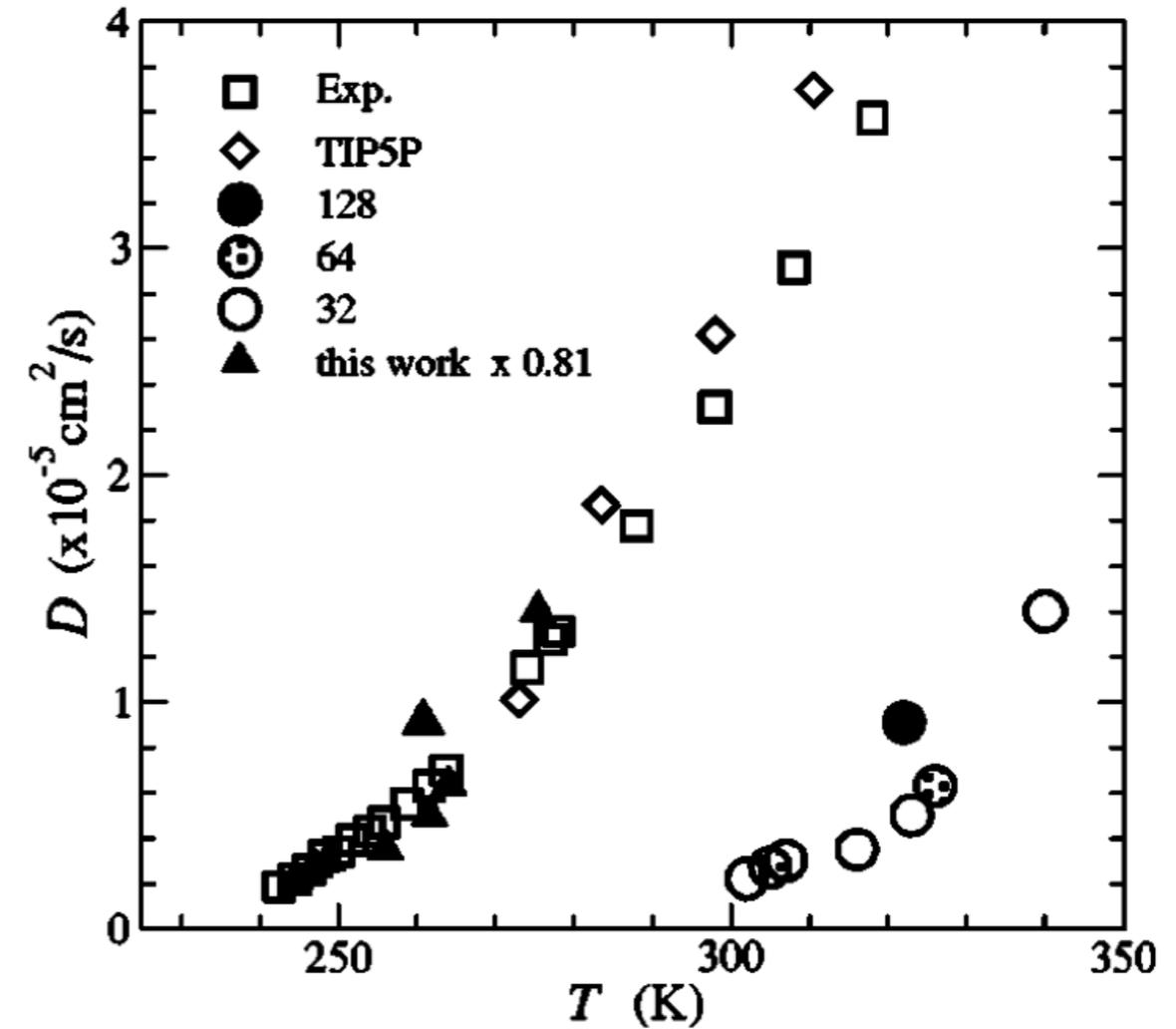
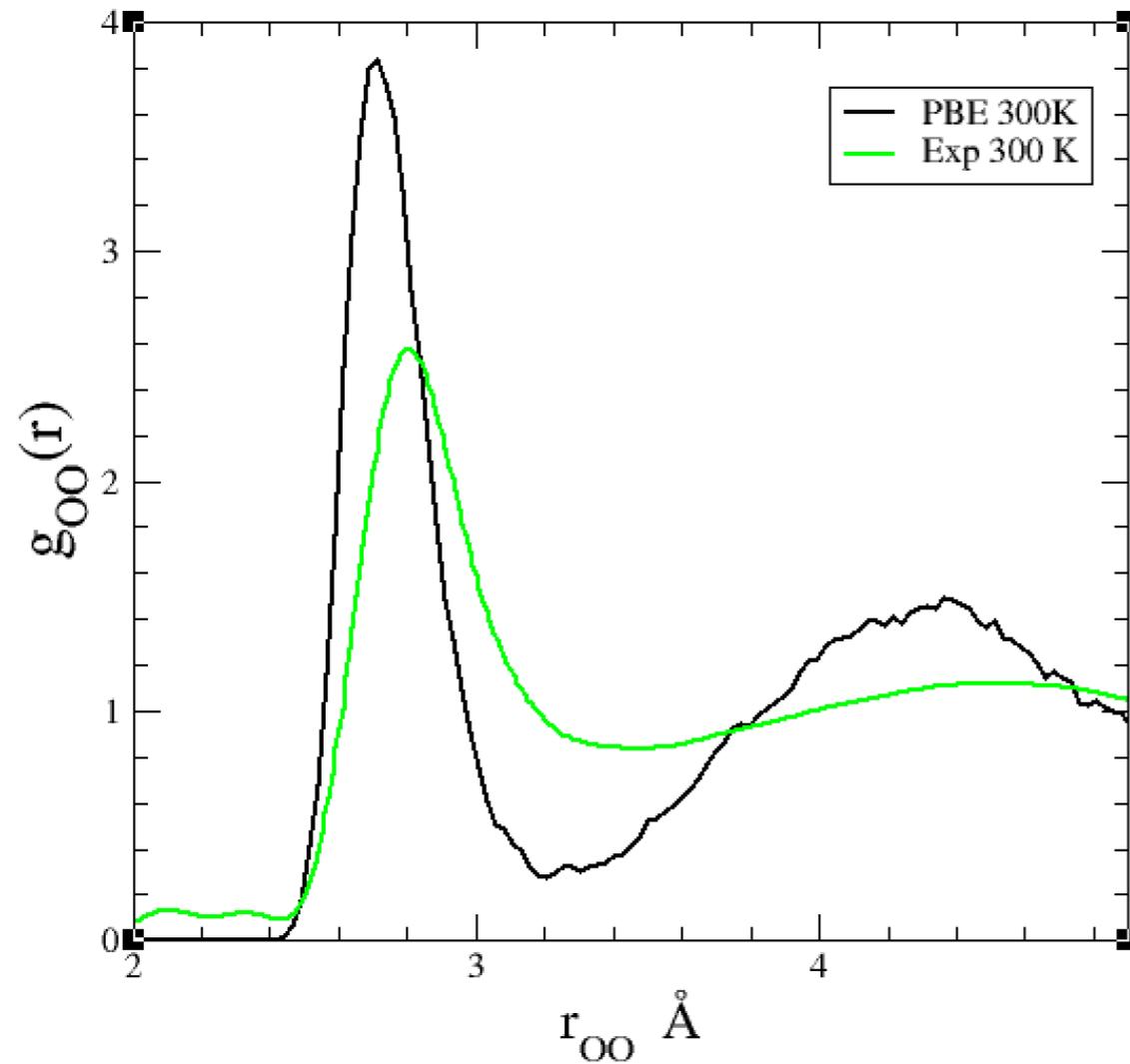
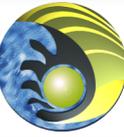
$$V_H(\mathbf{r}) = \int d^3\mathbf{r}' \frac{\rho_e(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$



$$\Psi(r_1, \dots, r_N) \rightarrow \psi_1(r_1) \psi_2(r_2) \dots \psi_N(r_N)$$



NON VDW DENSITY FUNCTIONALS



Exp: L. B. Skinner, C. Huang, D. Schlesinger, L. G. M. Pettersson, A. Nilsson, and C. J. Benmore, *J. Chem. Phys.* 138, 074506 (2013).

M.V Fernandez-Serra and E. Artacho. *J. Chem. Phys.* 121, 11136-11144 (2004).



VDW XC DENSITY FUNCTIONAL



Dion, Rydberg, Schröder, Langreth, and Lundqvist (DRSLL) PRL 92, 246401 (2004)

$$E_{xc} = E_x^{\text{GGA-revPBE}} + E_c^{\text{LDA}} + E_c^{nl}$$

$$E_c^{nl} = \frac{1}{2} \iint d\mathbf{r}_1 d\mathbf{r}_2 n(\mathbf{r}_1) n(\mathbf{r}_2) \Phi(q_1, q_2, r_{12})$$

$$q(n, \nabla n) = \left[1 + \frac{\epsilon_c^{\text{LDA}}(n)}{\epsilon_x^{\text{LDA}}(n)} - \frac{Z_{ab}}{9} \left(\frac{k_G}{2k_F} \right)^2 \right] k_F$$

$$Z_{ab} = -0.8491 \quad k_F = (3\pi^2 n)^{1/3} \quad k_G = \frac{\nabla n}{n}$$

Other proposals:

O.A.Vydrov and T.Van Voorhis, PRL 103, 063004 (2009)

J.Klimes, D.R.Bowler, and A.Michaelides, JPCM 22, 22201 (2010)

J.Wellendorff et al, Topics Catal. 53, 378 (2010)

K. Berland and Per Hyldgaard, PRB 89, 035412 (2014)

Non local kernel factorization:
Roman-Perez & Soler, PRL 2009

$O(N \log(N))$ cost



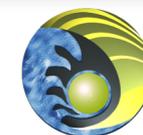
1.- DFT Liquid Water

vdW effects in Density

vdW effects in Structure

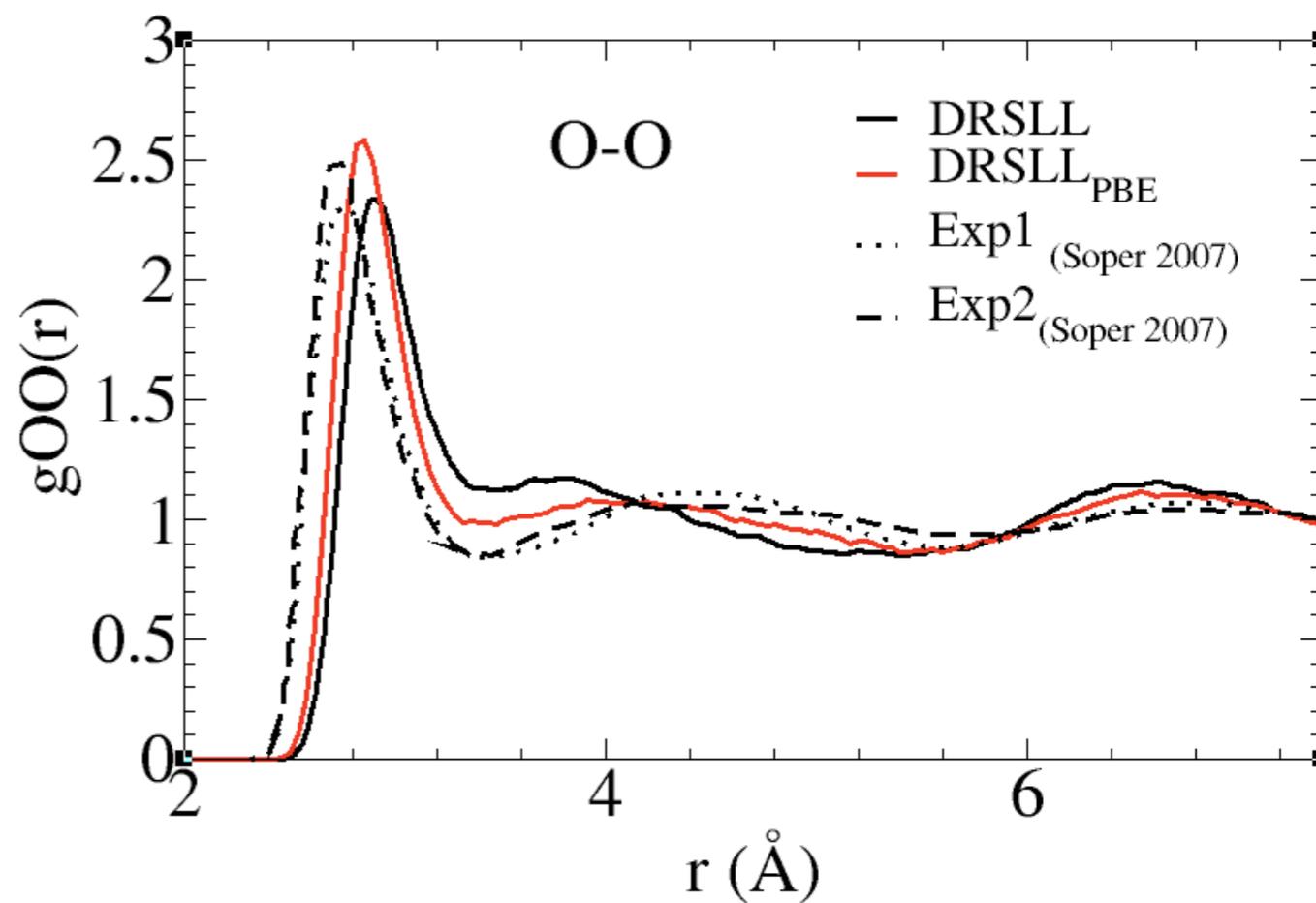
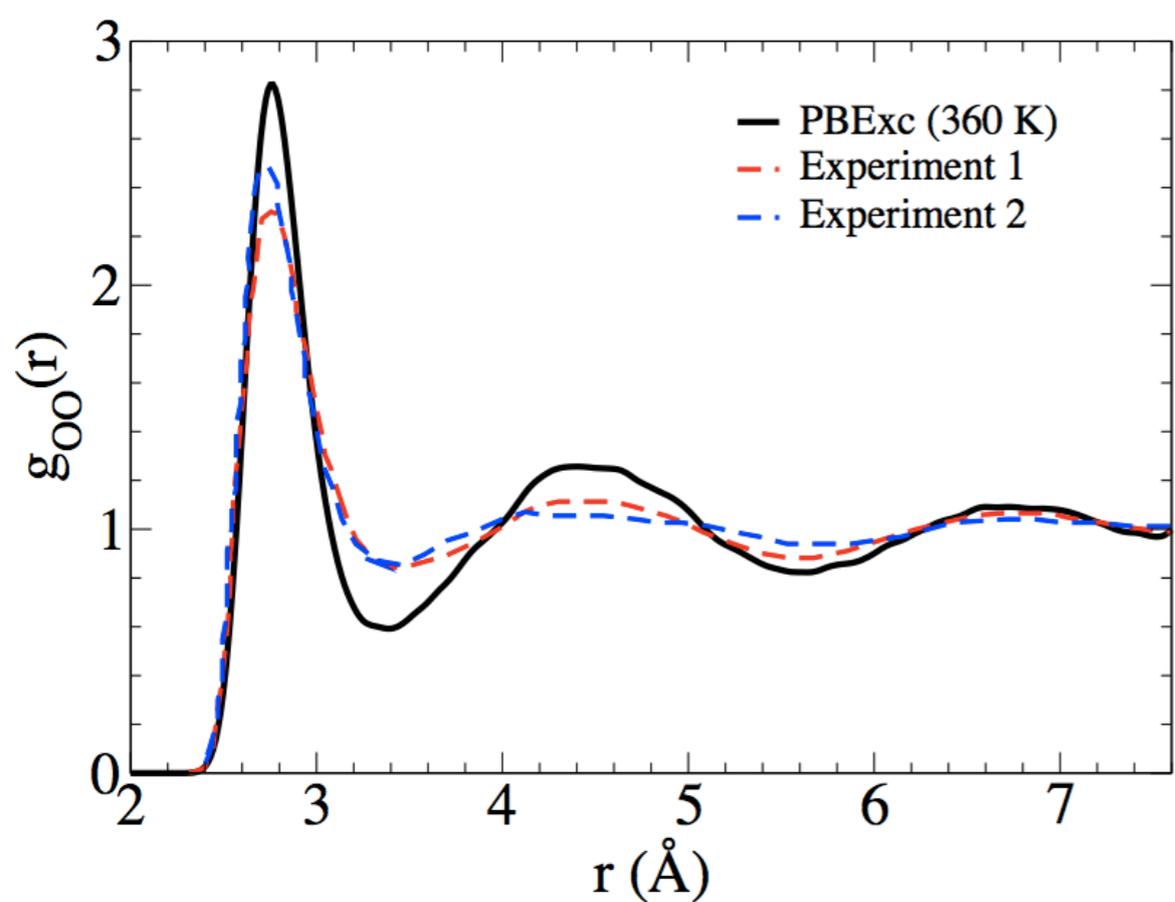


Pair distribution functions



semi-local XC

semi-local XC + non local C



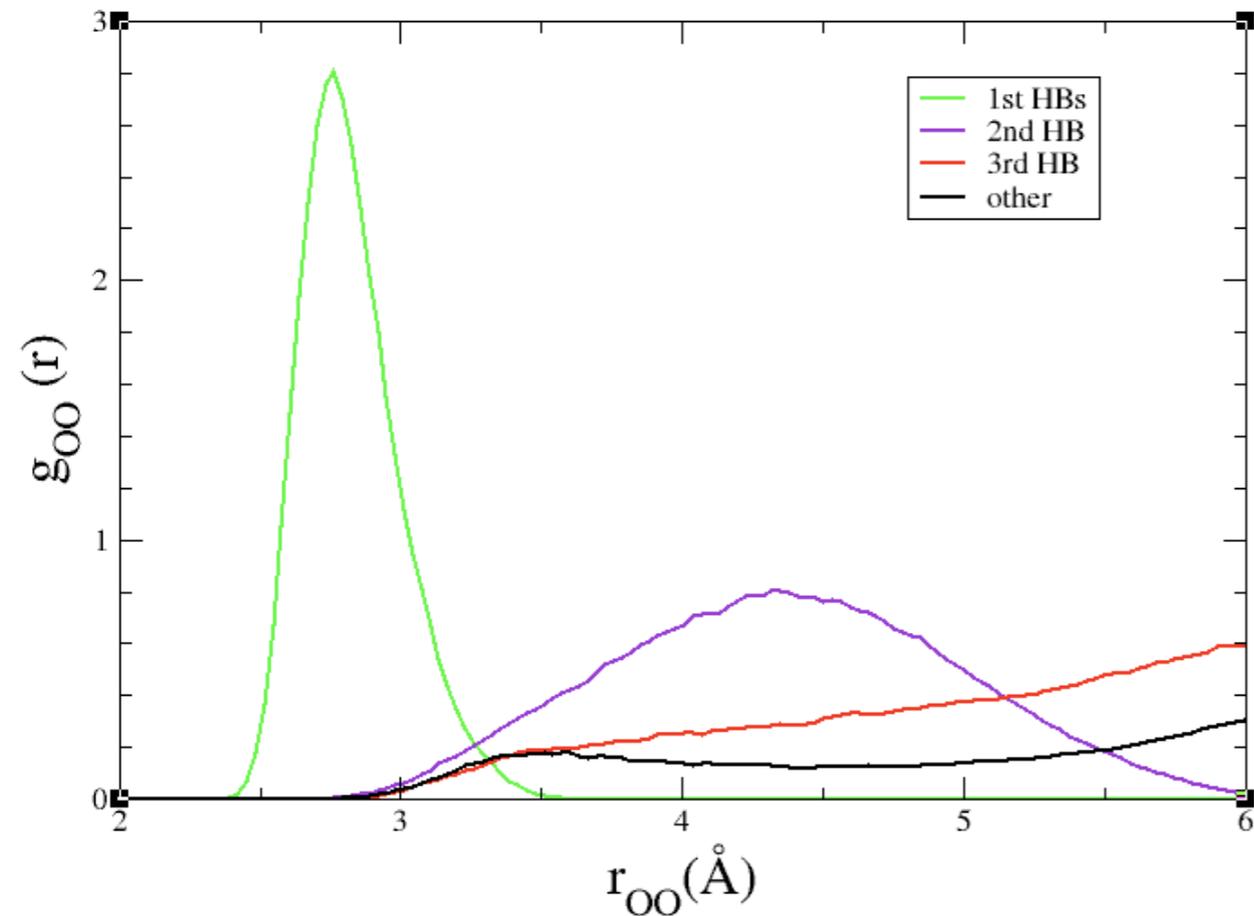
$$\rho = 1 \text{ g/cm}^3$$



H-bond network



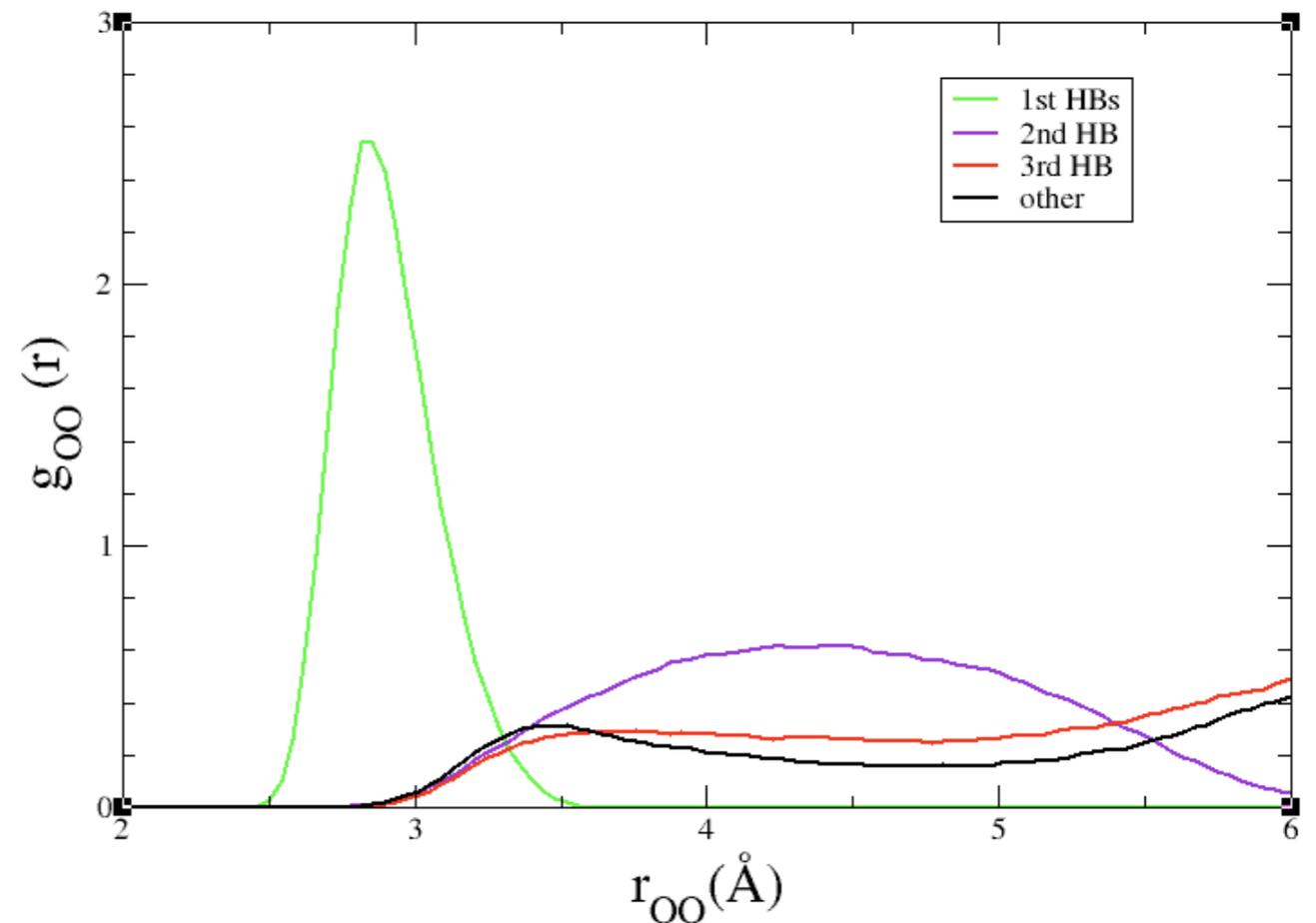
PBE ($\rho=1\text{ g/cm}^3$, 64 mol) PBE-VdW-DF, ($\rho=1\text{ g/cm}^3$, 64 mol)



3.81 H-bonds/molecule

$$\langle r_{\text{HB}} \rangle = 2.85$$

1.26 vdW bonds/molecule



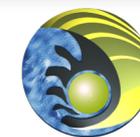
3.76 H-bonds/molecule

$$\langle r_{\text{HB}} \rangle = 2.93$$

1.99 vdW-bonds/molecule

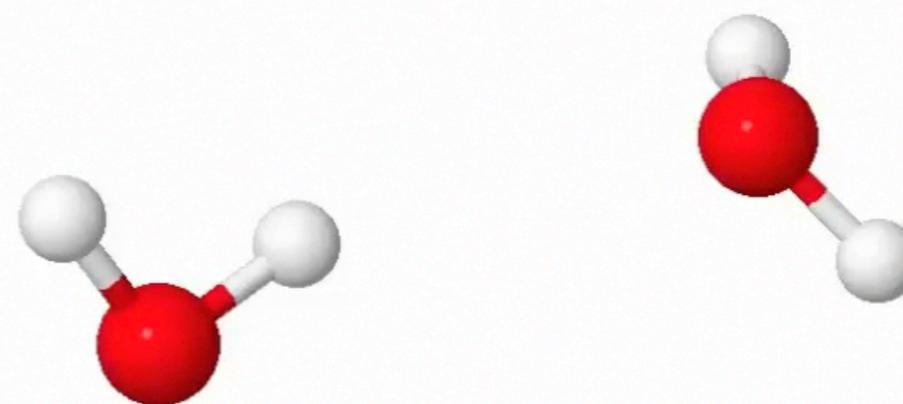
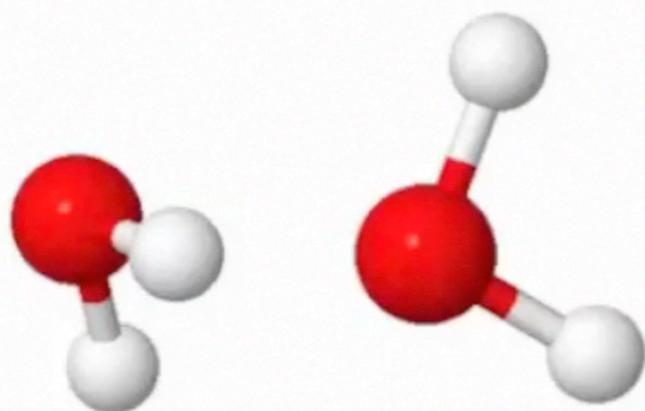


HBONDS AND VDW BONDS



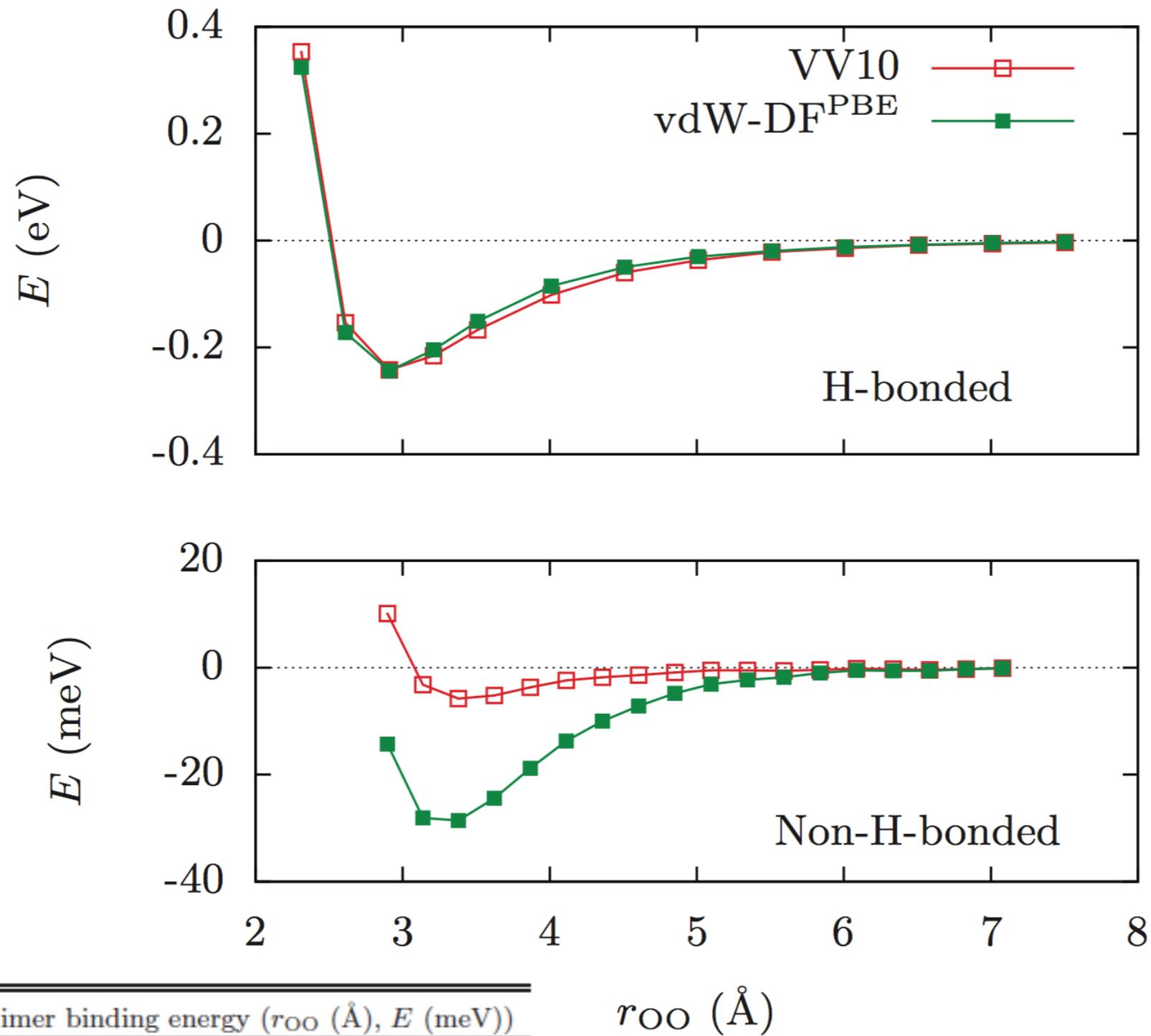
No vdW

with vdW





Dimer interaction energies



xc	Dimer binding energy (r_{OO} (Å), E (meV))	
	H-bonded	Non-H-bonded
VV10	(2.87, 247)	(3.39, 6)
vdW-DF ^{PBE}	(2.92, 245)	(3.27, 29)

r_{OO} (Å)

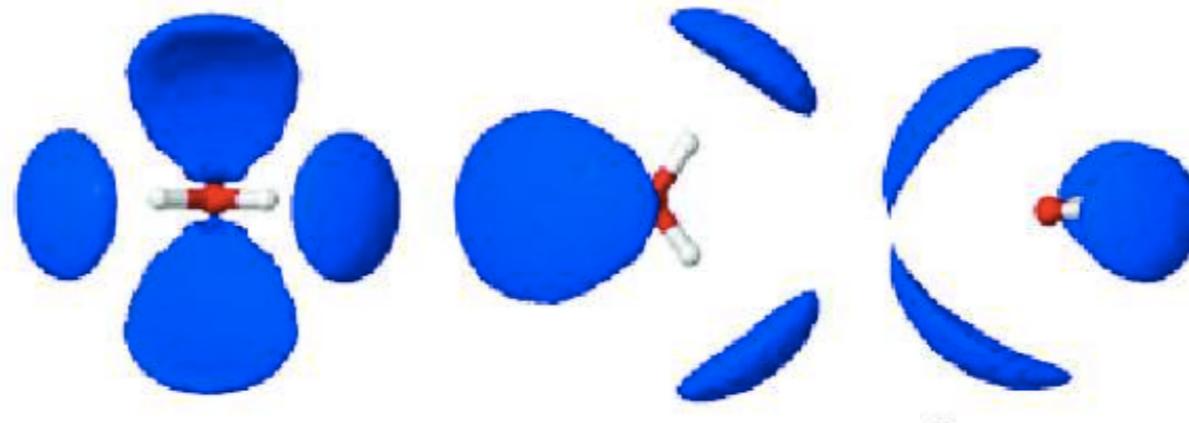


'Interstitial shell'



Spatial Distribution function

First shell
 $r = 2.8 \text{ \AA}$

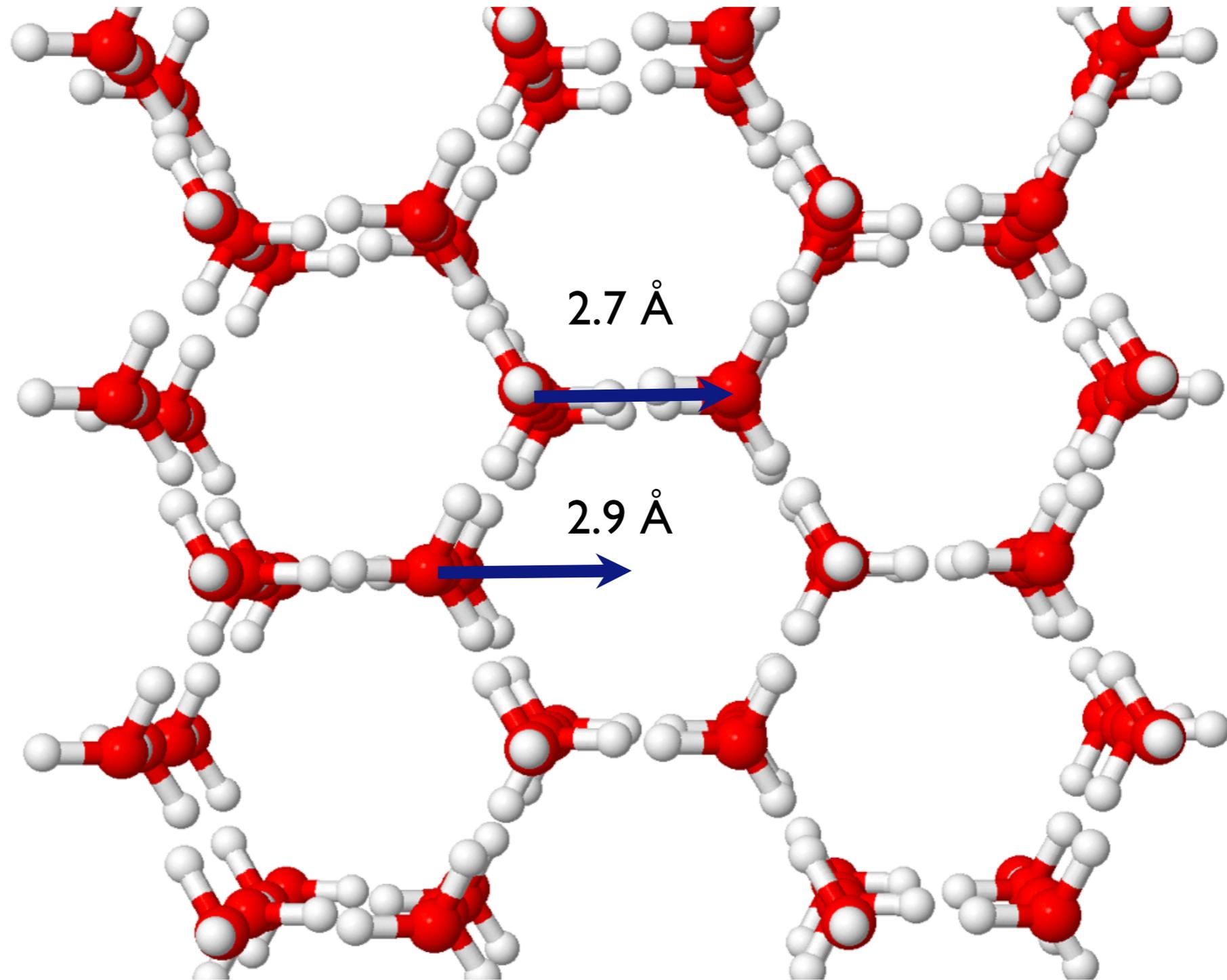


$r = 3.5 \text{ \AA}$



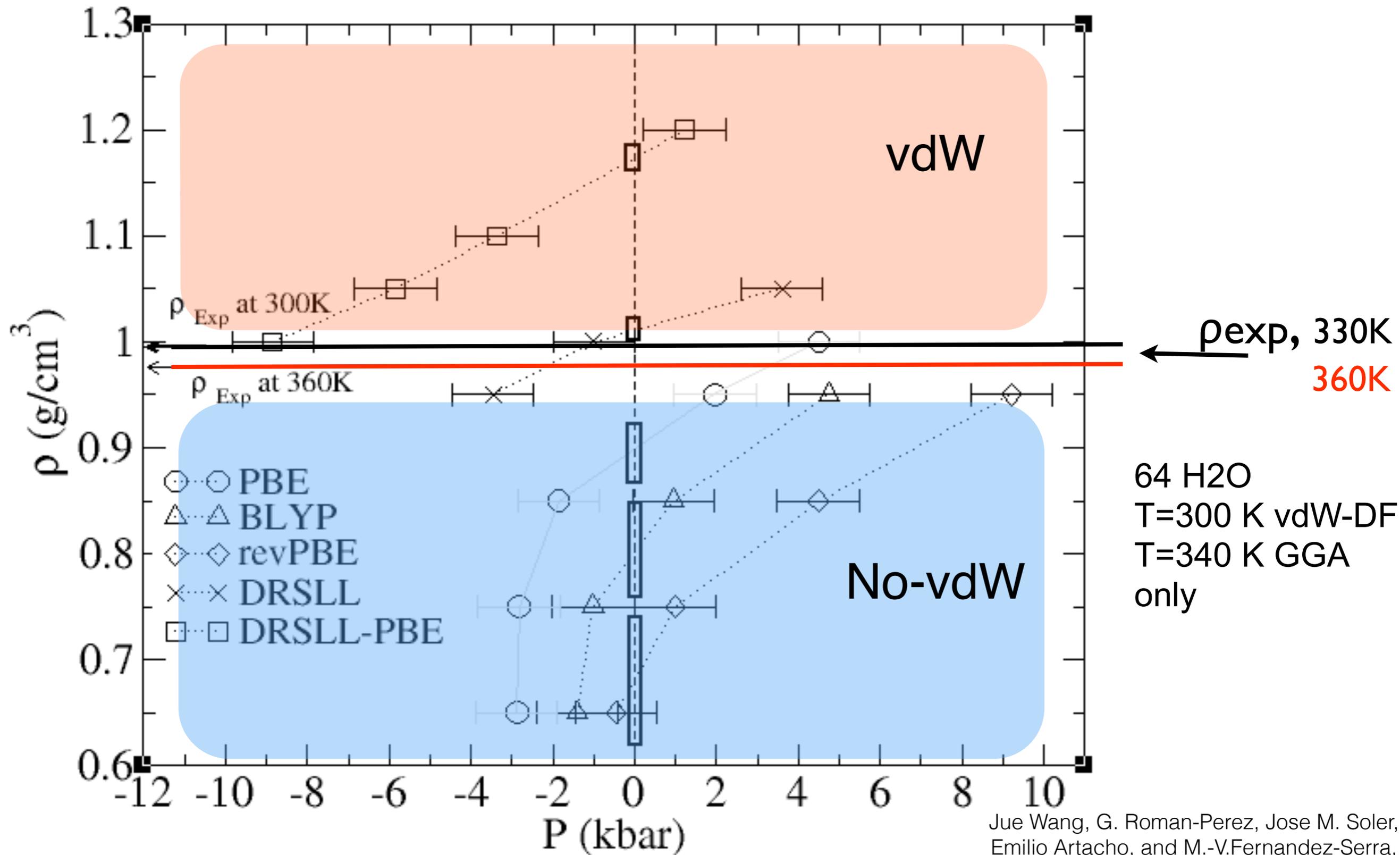
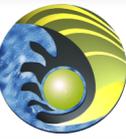


Interstitial sites: Ice-IH





Pressure-density curves



Jue Wang, G. Roman-Perez, Jose M. Soler, Emilio Artacho, and M.-V. Fernandez-Serra, J. Chem. Phys. 134, 024516 (2011).



- **Anomalous isotope effects in water and ice**

- Anomalous Nuclear Quantum Effects in Ice, B. Pamuk, J. M. Soler, R. Ramirez, C. P. Herrero, P.W. Stephens, P. B. Allen, and M.-V. Fernandez-Serra. Phys. Rev. Lett. 108, 193003 (2012).
- Quasi-harmonic approximation of thermodynamic properties of ice Ih,II, and III, R. Ramirez, N. Neuerburg, M.V. Fernández-Serra, and C. Herrero. J. Chem. Phys. 137, 044502 (2012)

- **Zero point effects and the phase diagram of Ice**

- Electronic and nuclear quantum effects on the ice XI/ice Ih phase transition. Pamuk, Philip B. Allen, and M.-V. Fernández-Serra Phys. Rev. B 92, 134105, 2015

- **Thermal conductivity of Ice: what can we learn about proton order?**

- Proton order and thermal conductivity of ice, J.T. Siebert, K. Chen, P. B. Allen and M.V Fernandez-Serra (in prep.)

- **Polar order in liquid water**

- Polar nanoregions in water - a study of the dielectric properties of TIP4P/2005, TIP4P2005f and TTM3F, Daniel C. Elton, M.V. Fernández-Serra, J. Chem. Phys, 140 , 124504 (2014).

- **Phonons in liquid water: Is water closer to a polar solid or to a polar liquid?**

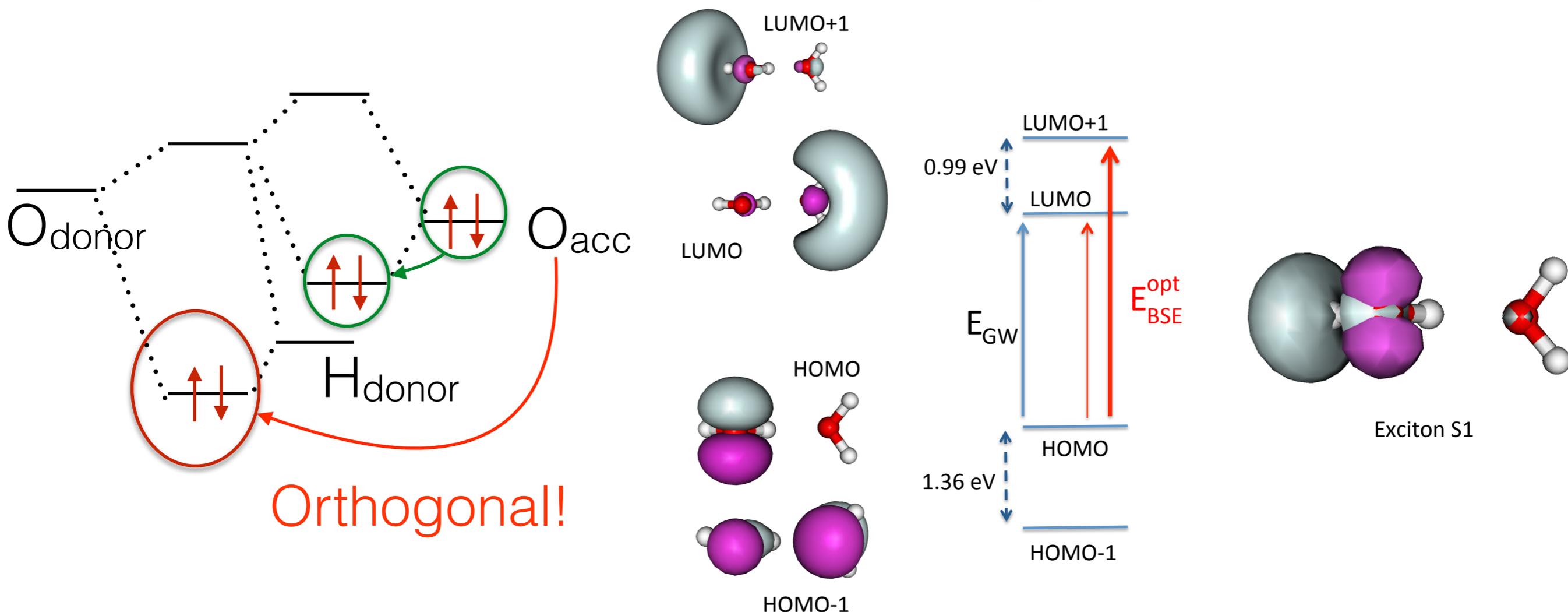
- The hydrogen bond network of water supports propagating optical phonon-like modes” arxiv.org/abs/1507.06363 Nature Communications 2016



H-BOND INTERACTION



Quantum-mechanical (electronic) component of the H-bond: intermolecular polarization, with mixing. Acceptor molecule's HOMO partially fills the LUMO of the donor molecule. This is the anti bonding of the covalent O-H bond, therefore weakening this covalent bond





COOPERATIVITY (MANY BODY EFFECTS)



- Many body effects have been shown to be critical to observe the existence of a LLPT in Giancarlo Franzese's many-body coarse grained model.
- The model reproduces most of the thermodynamical anomalies of water and presents a liquid-liquid phase transition ending in a critical point in the universality class of the two-dimensional (2D) Ising model.
- In the model these “quantum many body effects” are linked to the cooperative nature of the H-bond interaction.

Franzese, G., Marques, M. I. & Stanley, H. E. Intramolecular coupling as a mechanism for a liquid-liquid phase transition. *Phys. Rev. E* 67, 011103 (2003).

Franzese, G., Malescio, G., Skibinsky, A., Buldyrev, S. V. & Stanley, H. E. Generic mechanism for generating a liquid-liquid phase transition. *Nature* 409, 692 (2001).



COOPERATIVITY (MANY BODY EFFECTS)



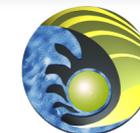
Different Physical Sources

Manifestation

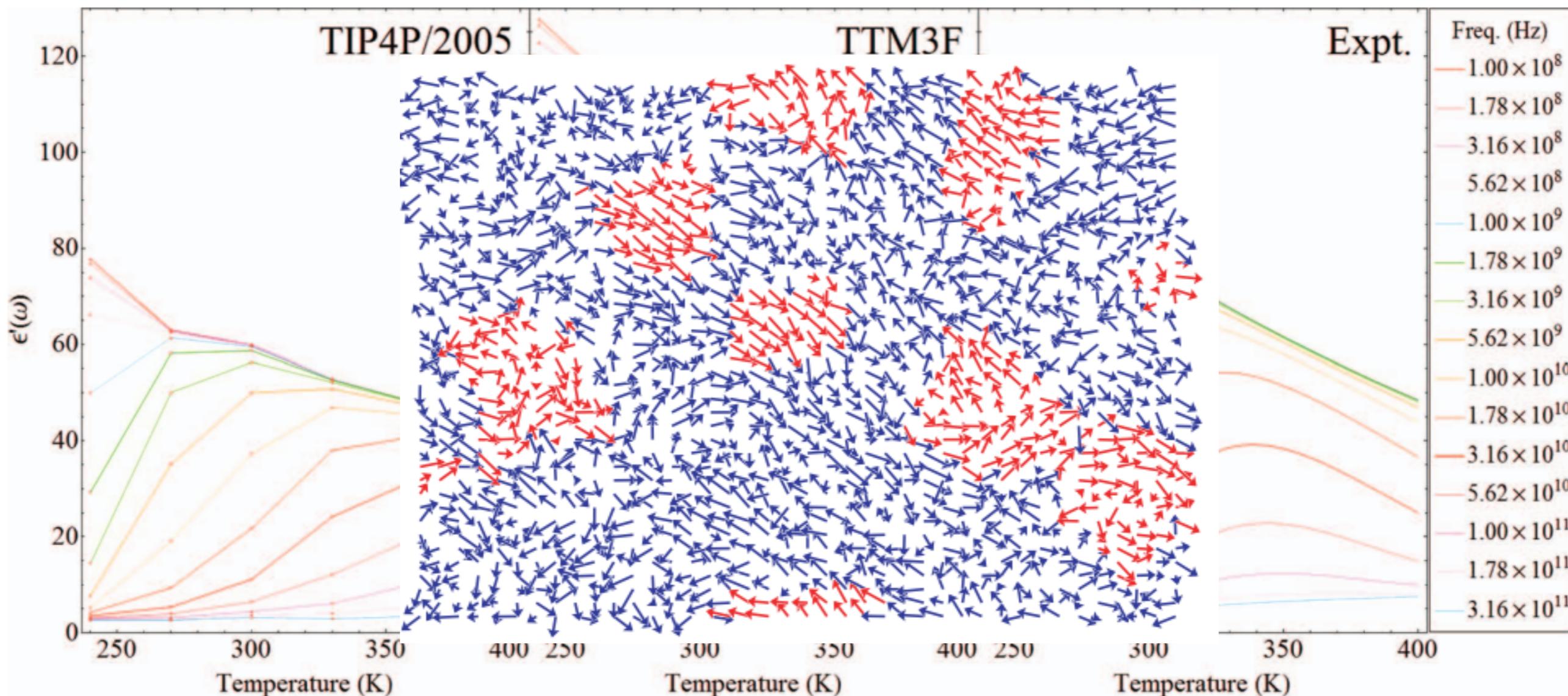
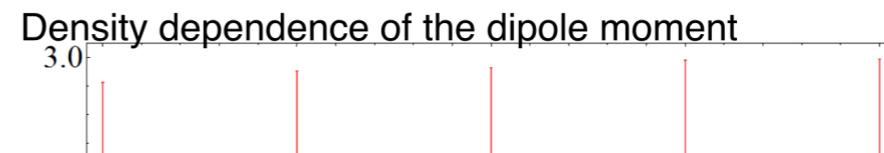
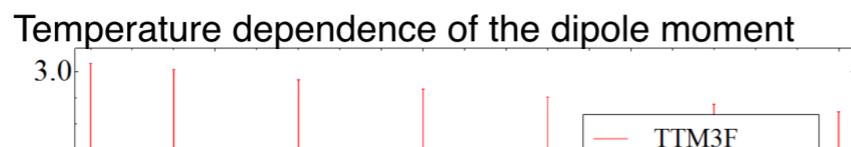
- | | | |
|--|---|--|
| 1. Intramolecular polarization (electrostatic) |  | 1. Dipole enhancement, D(T,P) |
| 1. Intermolecular polarization (small charge transfer) |  | 2. Anticorrelation OH-OO distances and intra-inter vibrational frequencies |
| 2. Long Range electrostatic effects |  | 3. Locally polarized regions (polar nano regions), LO-TO splitting and propagating phonons |
| 3. Nuclear Quantum Effects |  | 4. Enhance all previous effects |



Intramolecular Polarization



Both and electronic and ionic contribution
Present in all flexible and polarizable force fields



- TTM3F better reproduces $\Delta\epsilon(U) / \Delta I$ and $\Delta\epsilon(U) / \Delta V$
- Change in entropy in E field is proportional to $\Delta\epsilon(0) / \Delta T$

öhlich, *Theory of Dielectrics* (Oxford Uni. Press, 1949).

Daniel C. Elton, M. V. Fernández-Serra, J. Chem. Phys., 140, 124504 (2014).



DIPOLE LONG RANGE ORIENTATIONAL ORDER

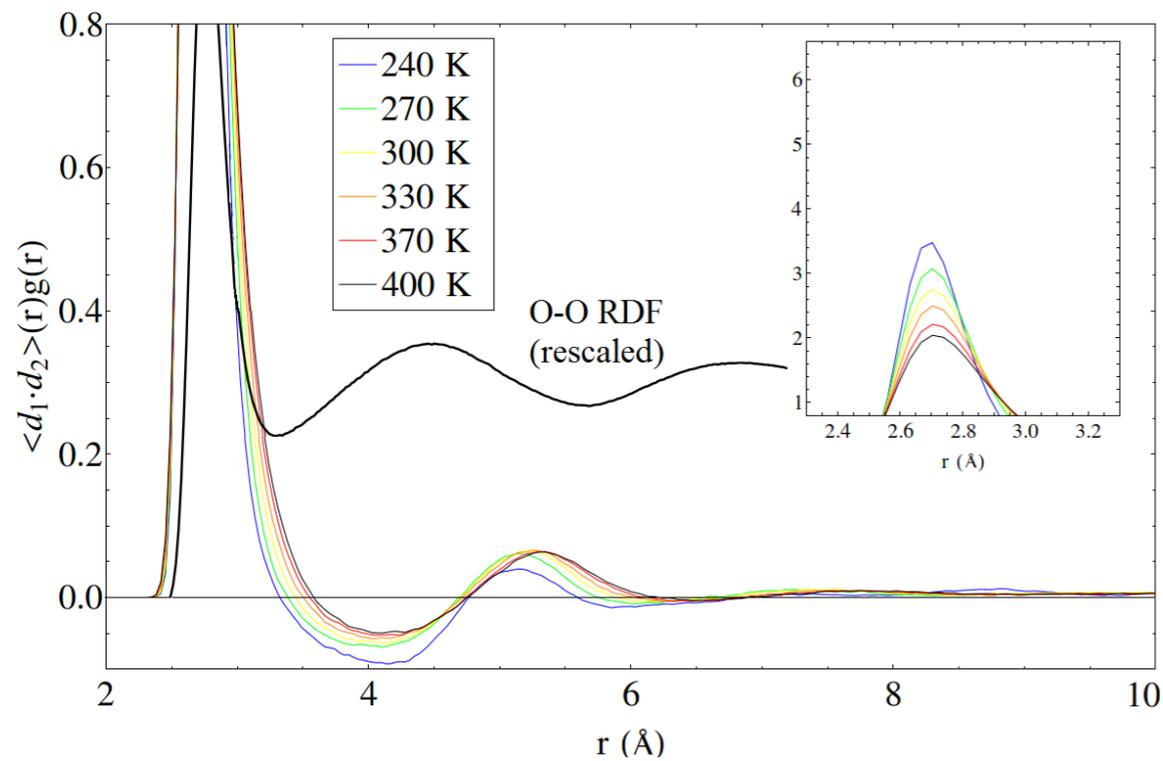


“dip-dip correlation function” – gives average angle, weighted by density of molecules (O-O RDF)

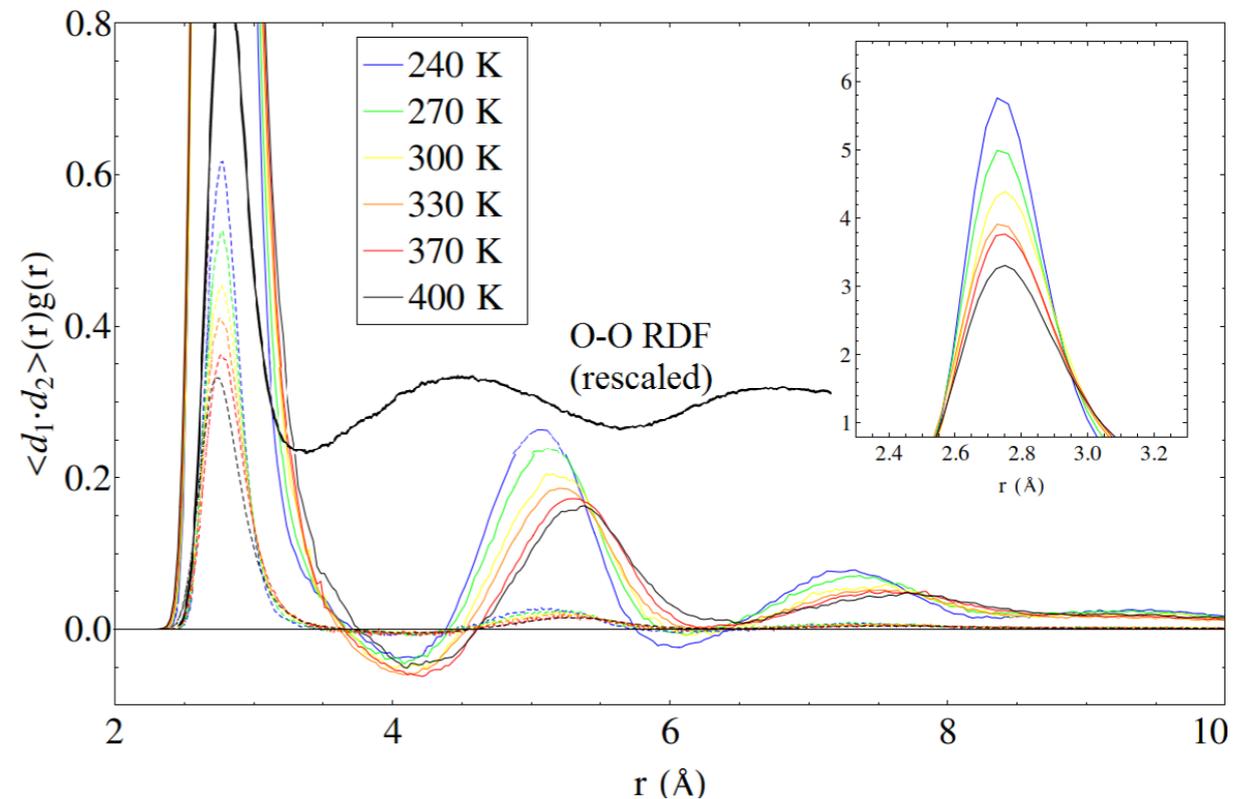
$$\phi(r) = \frac{1}{N_{\text{gas}}(r)} \sum'_{i,j} \mu_i \cdot \mu_j \quad r < r_{ij} < r + \delta r$$

$$= \langle \mu_1 \cdot \mu_2 \rangle(r) g_{\text{OO}}(r)$$

TIP4P/2005f dip-dip correlation function



TTM3F dip-dip correlation function

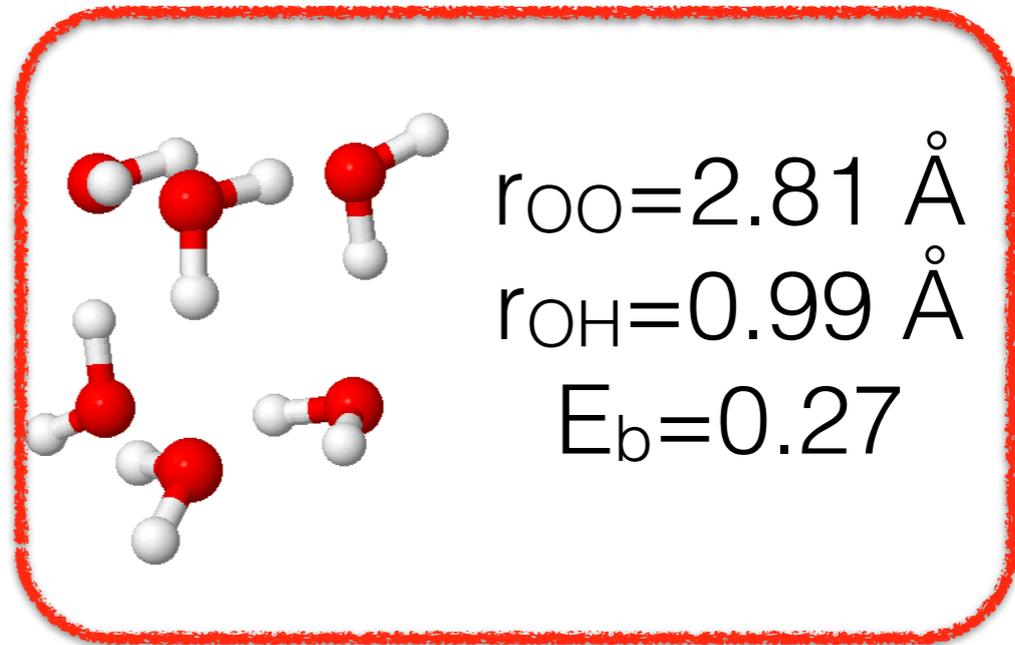
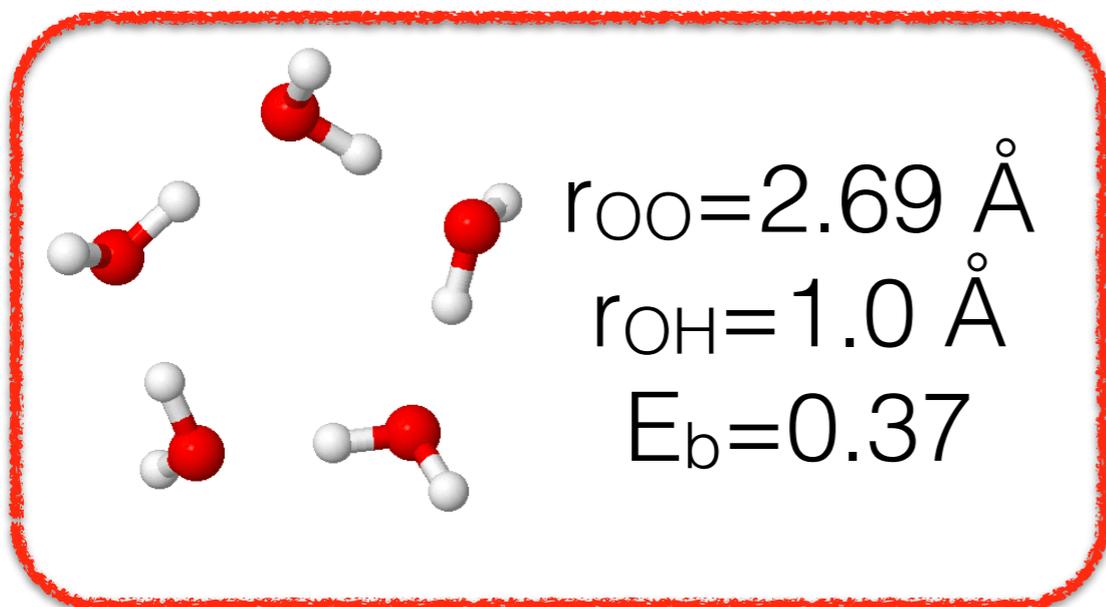
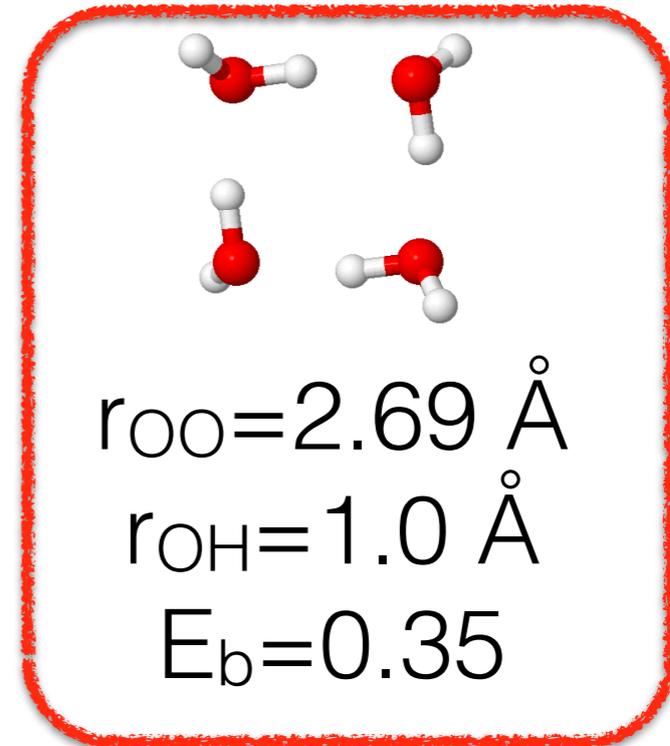
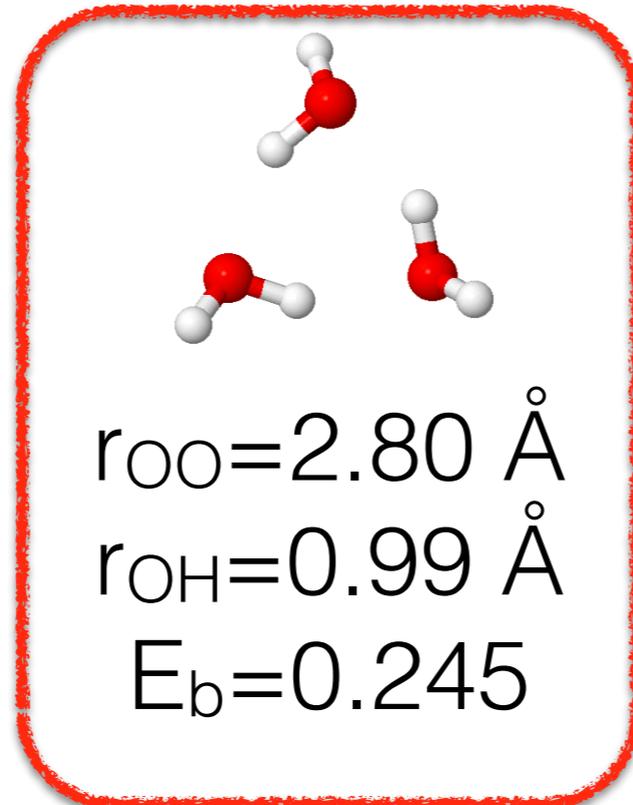
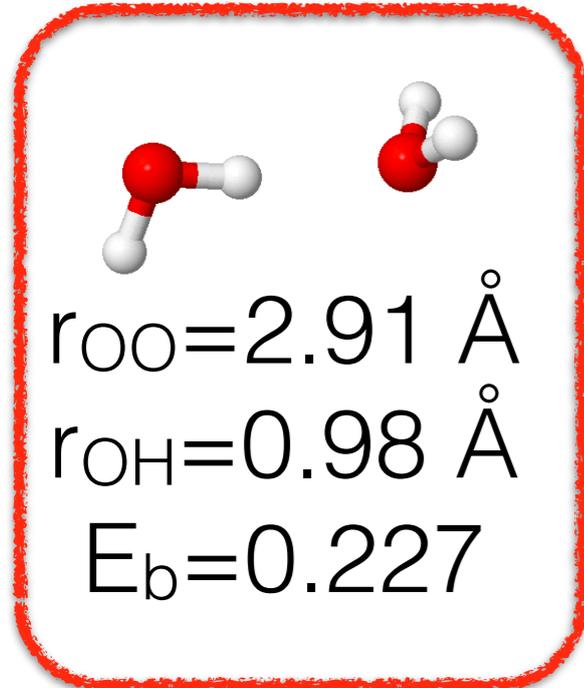


Polar Nanoregions (PNRs): Cooperative rearranging regions
Same behavior as Relaxor Ferroelectric materials

Daniel C. Elton, M. V. Fernández-Serra, J.
Chem. Phys, 140 , 124504 (2014).

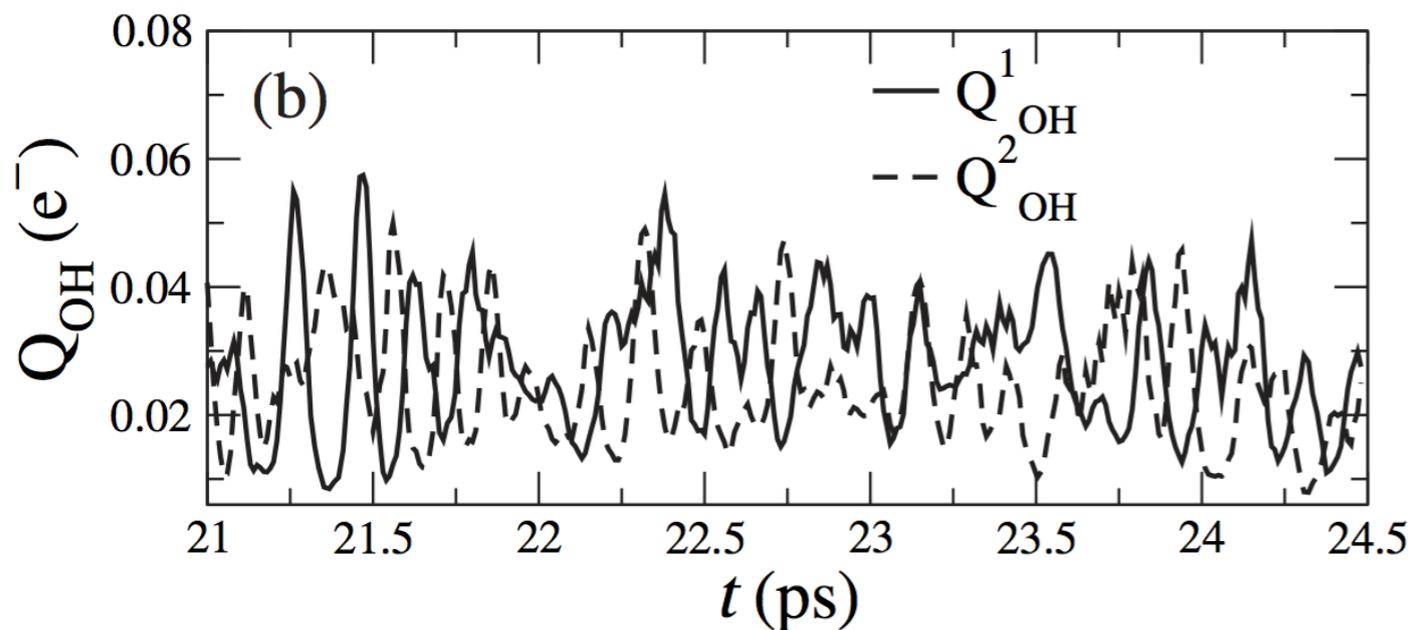


COOPERATIVITY IN HBOND CHAINS





The main source of enhanced Hbond interaction in liquid water are the intra- and inter-molecular polarization effects. In addition, there is an observed electronic beat which alternates between strong and weak Hbonds, that can be delocalized along a chain of several Hbonded molecules.



M.V Fernandez-Serra and E. Artacho.
Phys. Rev. Lett, 96, 016404 (2006).

T. D. Kuhne, R. Z. Khaliullin.

“Electronic signature of the instantaneous asymmetry in the first coordination shell of liquid water.”

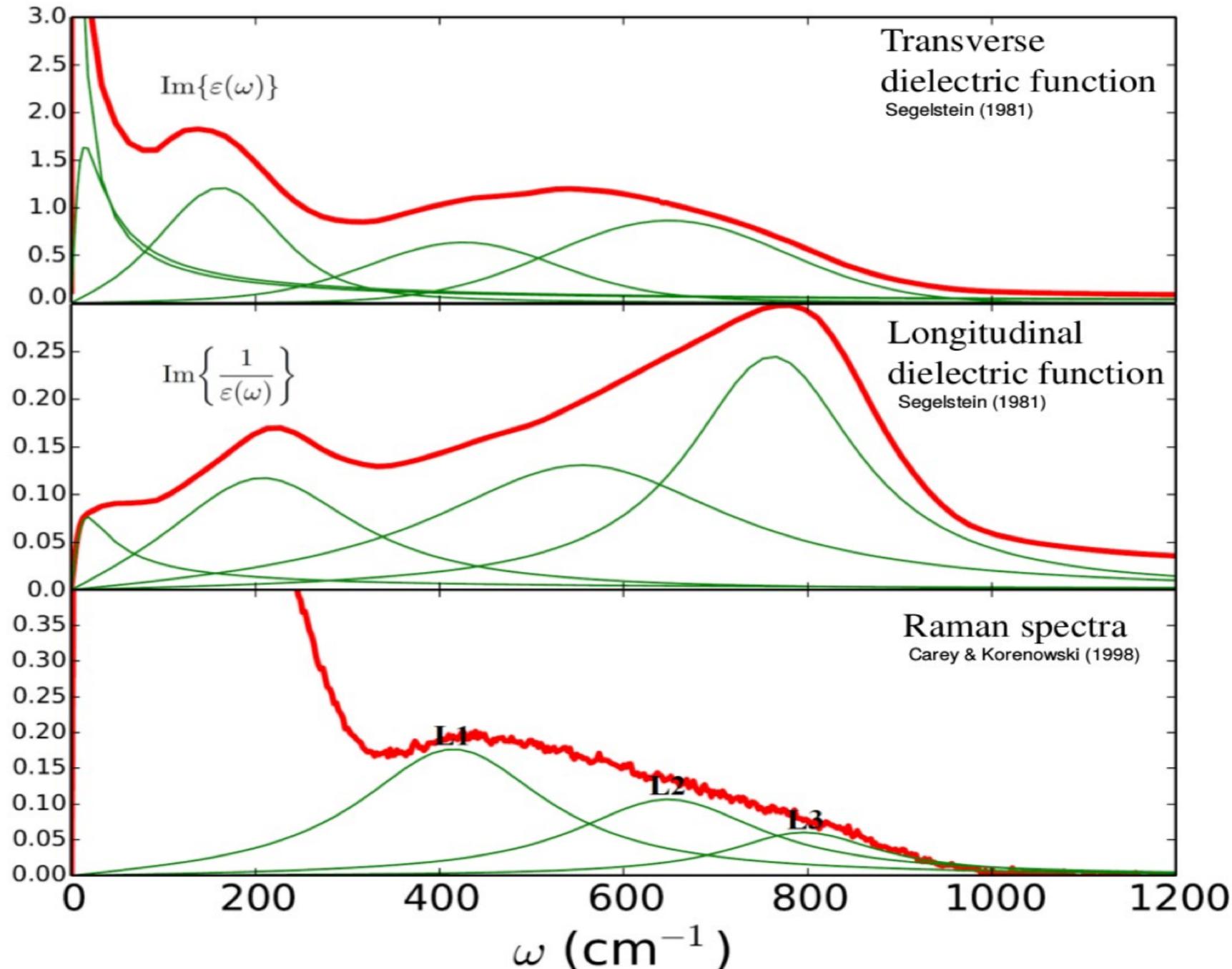
Nature Communications, 4, 1450 (2013)



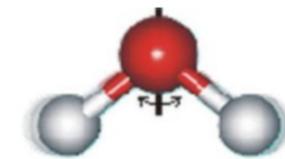
VIBRATIONAL PROPERTIES OF WATER



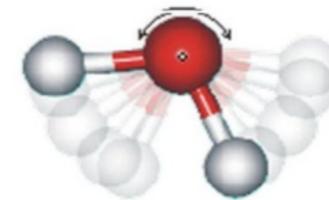
Problematic peak assignments



Librational motions of a single water molecule

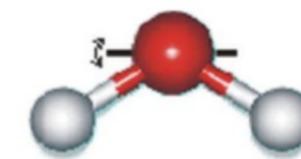


L1 = "twisting"



L2 = "rocking"

IR active



L3 = "wagging"

IR active

"The hydrogen bond network of water supports propagating optical phonon-like modes",

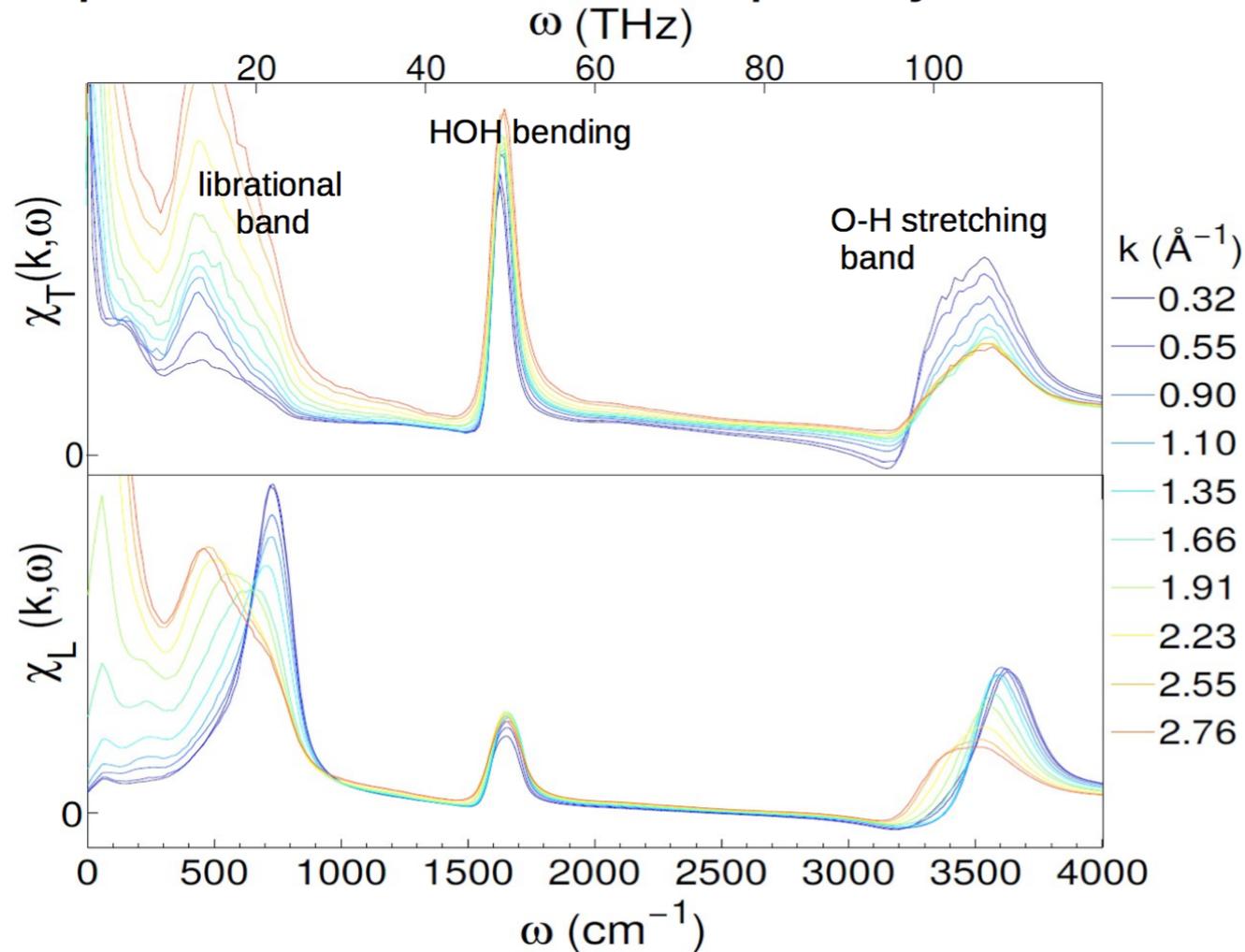
D. Elton and M. V. Fernández-Serra, Nature Communications 7, Article number: 10193, (2016)



VIBRATIONAL PROPERTIES OF WATER



k -dependent dielectric susceptibility of TTM3F



$$\phi(k, t) = \left\langle \sum_i \mathbf{p}_i(k, 0) \cdot \sum_j \mathbf{p}_j(k, t) \right\rangle$$

$$\chi_{L/T}(k, \omega) = \chi_{L/T}(k, 0) \int_0^\infty \dot{\Phi}_{L/T}(k, t) e^{i\omega t} dt$$

$$\chi_L(k \rightarrow 0, \omega) = 1 - \frac{1}{\varepsilon(\omega)}$$

$$\chi_T(k \rightarrow 0, \omega) = \varepsilon(\omega) - 1$$

$$\varepsilon'(\omega) = n^2(\omega) - k^2(\omega)$$

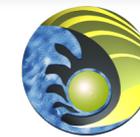
$$\varepsilon''(\omega) = 2n(\omega)k(\omega)$$

“The hydrogen bond network of water supports propagating optical phonon-like modes”,

D. Elton and M. V. Fernández-Serra, Nature Communications 7, Article number: 10193, (2016)

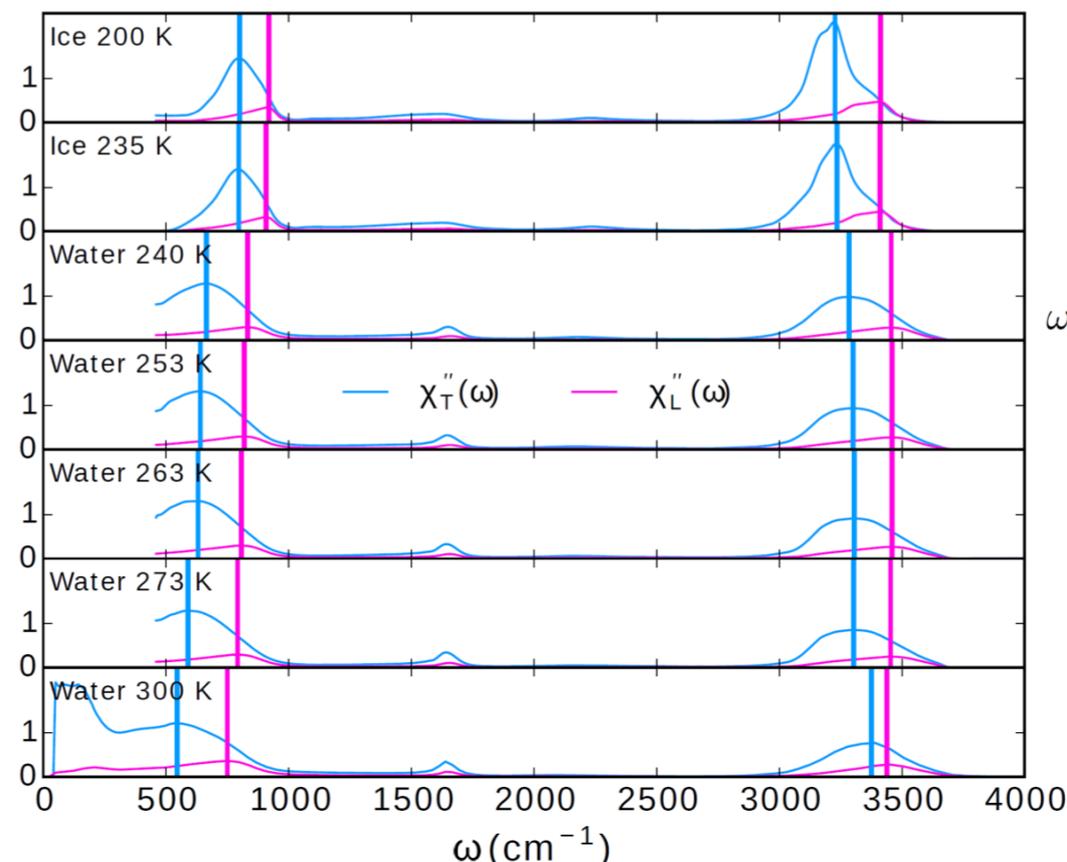
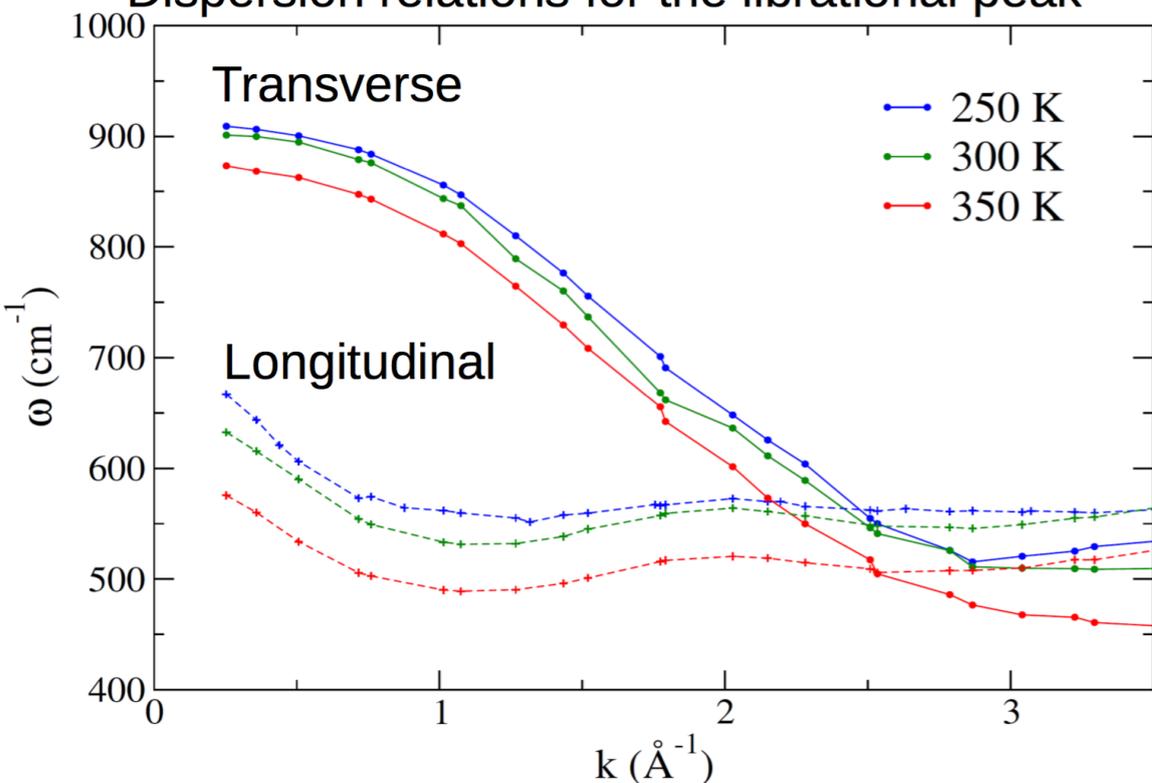


VIBRATIONAL PROPERTIES OF WATER



LO-TO splitting

Dispersion relations for the librational peak



Relation to crystal structure:

$$\omega_{Lk}^2 - \omega_{Tk}^2 = \frac{4\pi C}{3v} \left(\frac{\partial \mu}{\partial Q_k} \right)^2$$

v = volume per unit cell

C = depends on crystal structure

Table 1 | Resonance frequencies and lifetimes.

Model	Temp	ω_{LO}	τ_{LO}	ω_{TO}	τ_{TO}	$\omega_{LO} - \omega_{TO}$
TIP4P/2005f	250	905	0.38	667	0.23	233
	300	900	0.44	632	0.18	268
	350	871	0.34	574	0.18	297
	400	826	0.25	423	0.17	400
3*TTM3F	250	757	0.49	496		261
	300	721	0.44	410		311
	350	710	0.20	380		330
expt ³⁴	253	820		641		179
expt ³⁵	300	759		556		203

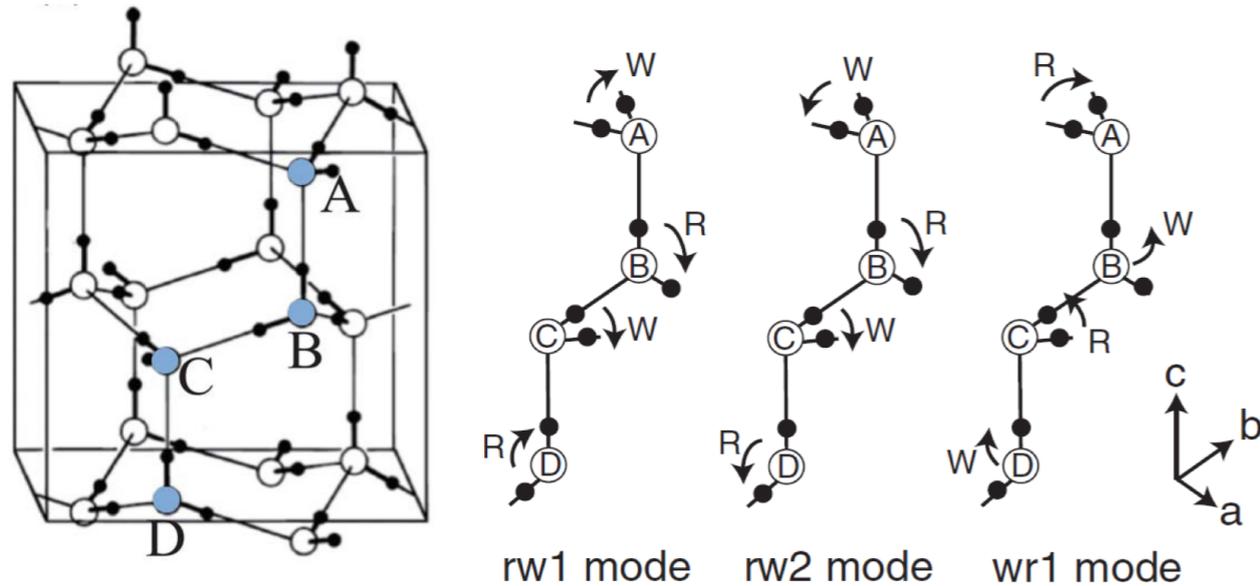
Frequencies are given in cm^{-1} and lifetimes in ps. The values from simulation were computed at the smallest k in the system. The experimental values are based on the position of the max of the band and therefore only approximate.



PHONONS IN ICE AND RANGE OF PROPAGATION

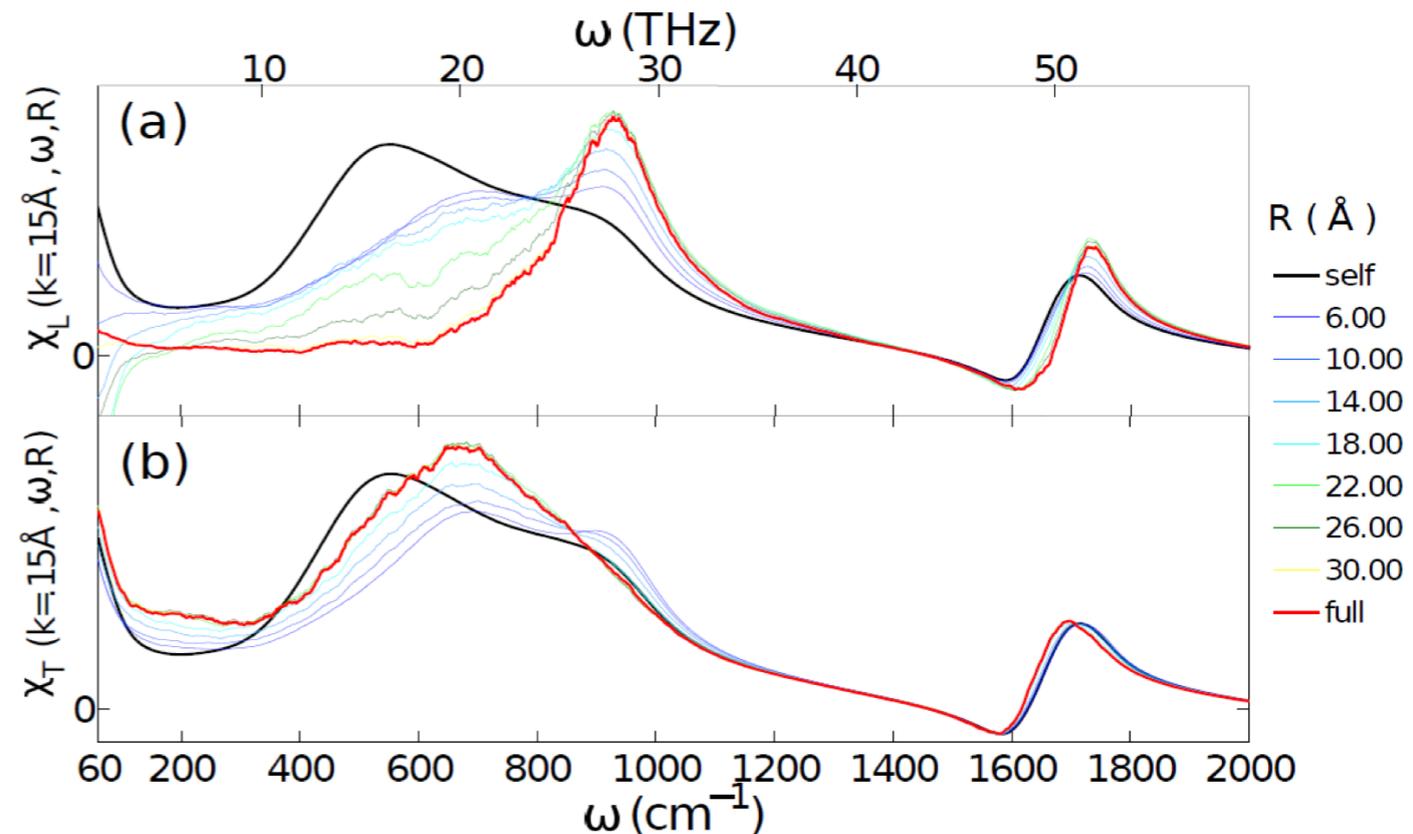


Corresponding librational modes in ice XI



Ref: Iwano et al. *J. Phys. Soc. Jap.* **79**, 063601 (2010).

Distance decomposition of the spectra



- The dynamics of water are closer to that of ice than previously thought.
- We reassign librational & stretching spectral peaks to phonon-like modes.
- The librational modes correspond to coupled rocking and wagging motions
- lifetimes of $\sim .1 - .5$ ps
- propagation distances of up to 2 nm
- modes exhibits LO-TO splitting
- LO-TO splitting is a novel probe of local structure



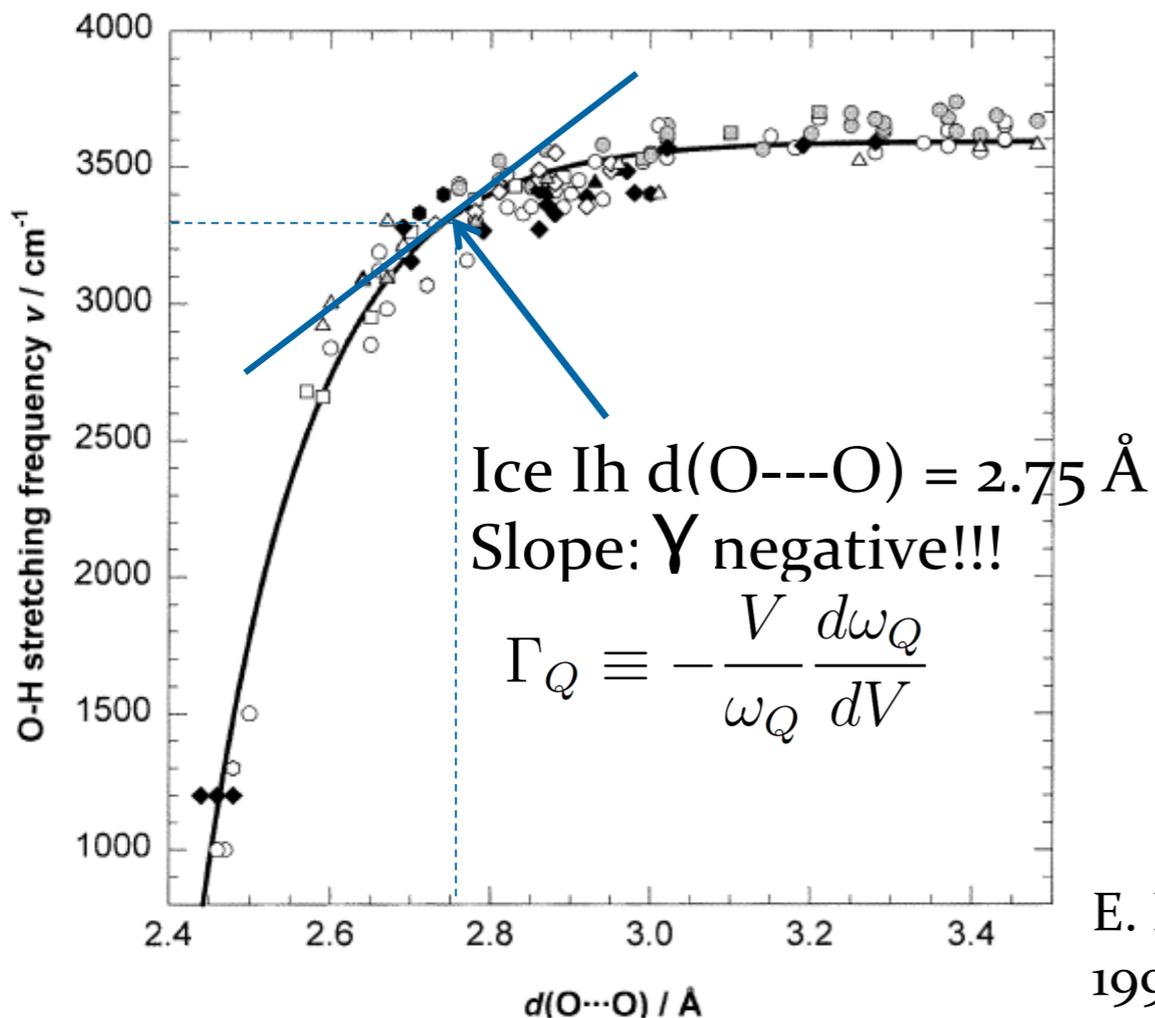
**Example of DFT success:
Anomalous Isotope effects
in ice and water**



OH--O ANTI-CORRELATION EFFECT



- Well known effect in water and ice
- Common in Hbonded systems.



E. Libowitzky,
1999

C. J. Burnham, G. F. Reiter, J. Mayers, T. Abdul-Redah, H. Reichert and H. Dosch, **On the origin of the redshift of the OH stretch in Ice Ih: evidence from the momentum distribution of the protons and the infrared spectral density**, Phys. Chem. Chem. Phys., 2006, 8, 3966

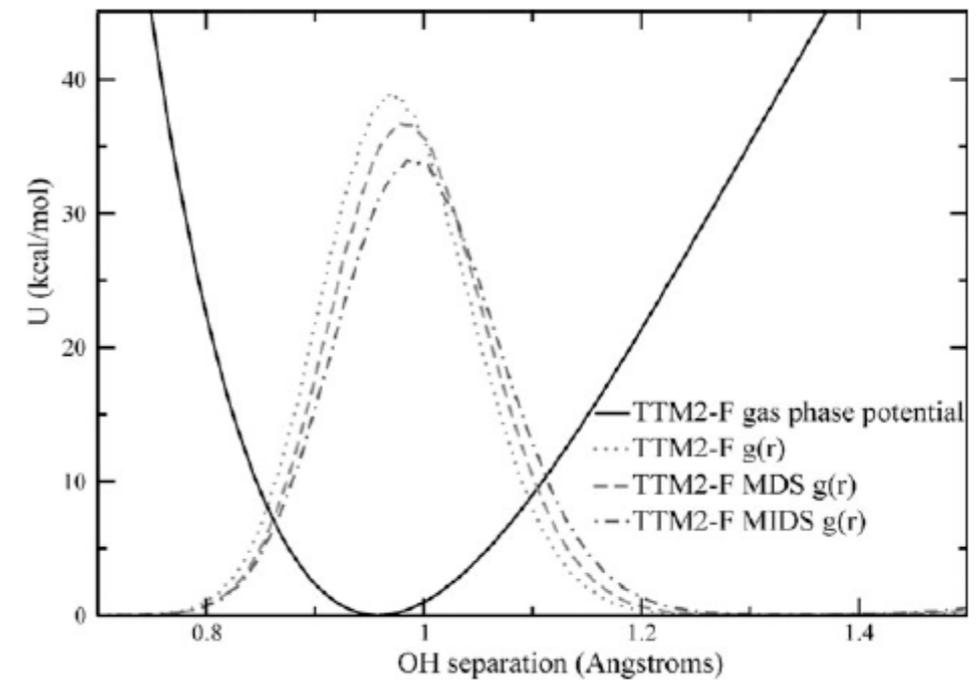


Fig. 1. Plot of the $d(\text{O}\cdots\text{O})$ – frequency correlation; open symbols represent straight H bonds, shaded symbols mark bent H bonds, and filled ones denote copper compounds; circles – silicates, squares - (oxy)hydroxides, hexagons – carbonates, diamonds – sulfates, triangles – phosphates and arsenates; the regression curve was calculated for distances $< 3.5 \text{ \AA}$ ($n = 124$) in the form $\nu = 3592 - 304 \cdot 10^9 \cdot \exp(-d/0.1321)$, $R^2 = 0.96$



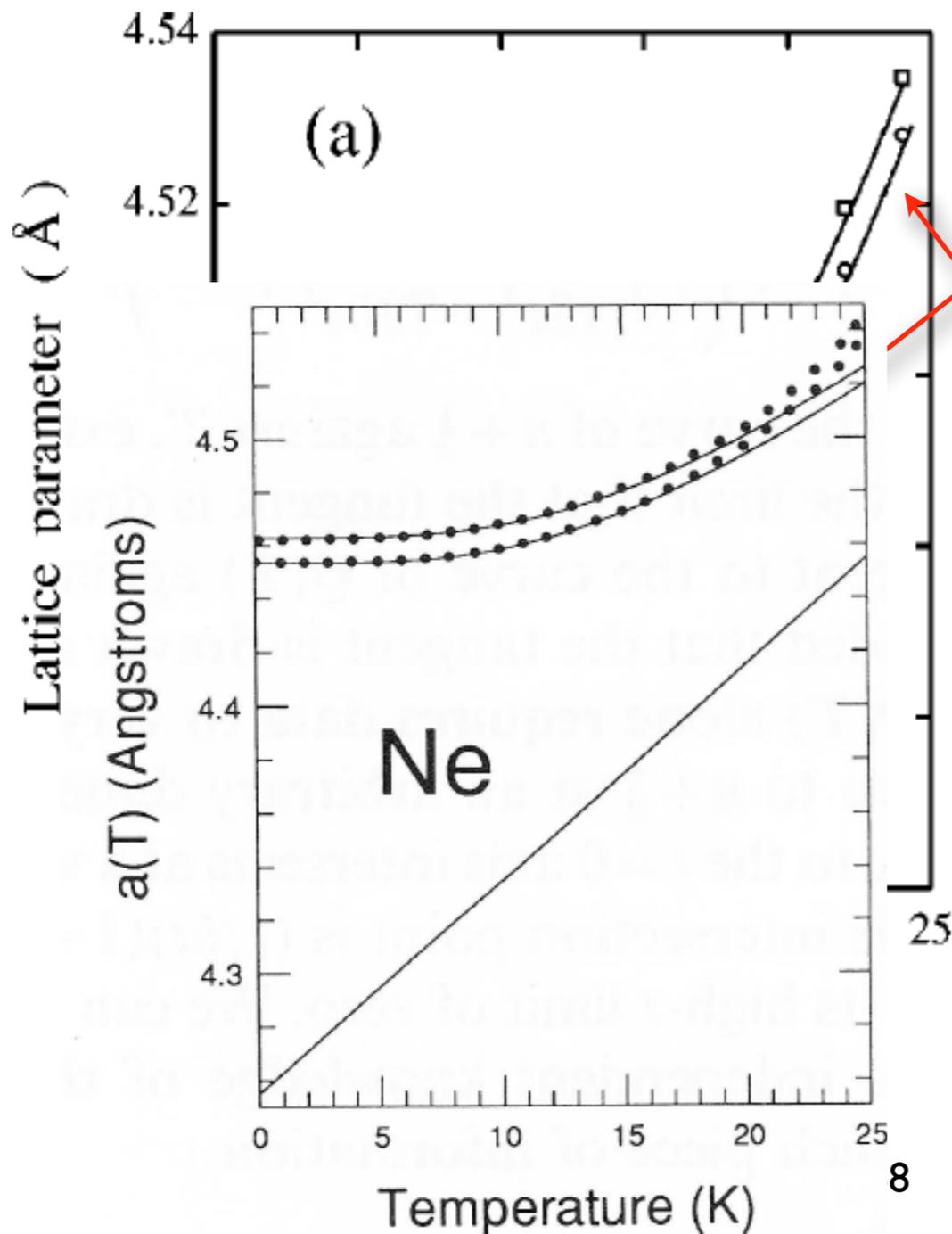
Isotope shift of lattice constant



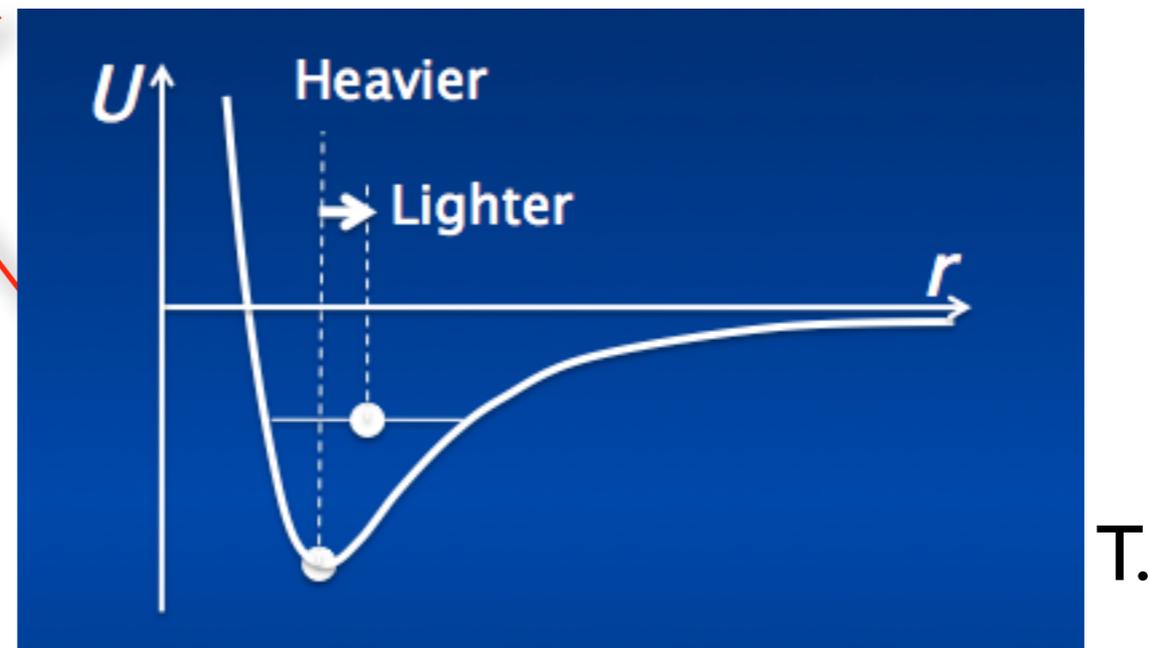
Bolz and Mauer, 1962

Batchelder, Losee, and Simmons, 1968

Phil Allen, 1994, Herrero 96



Heavy neon (^{22}Ne) occupies 0.19% less space than light neon (^{20}Ne) at $T=0$.



Extrapolated bare lattice constant $a=4.255\text{Å}$
~12% zero point volume expansion



Isotope effects in ice: experiments

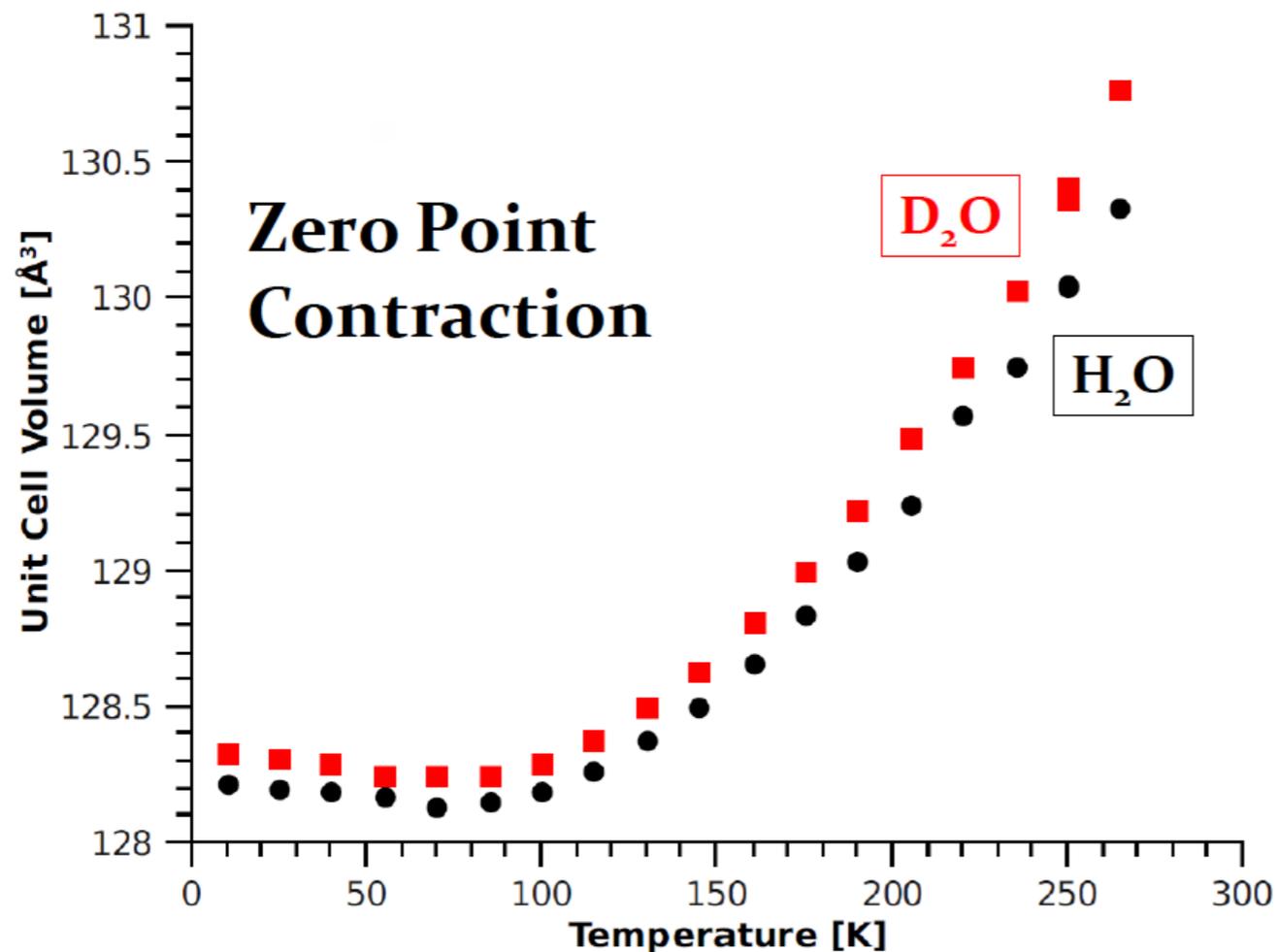


K. Röttger, A. Endriss, J. Ihringer, S. Doyle and W. F. Kuhs

Lattice constants and thermal expansion of H₂O and D₂O ice Ih between 10 and 265K

Acta Cryst. (1994). B50, 644-648

Acta Cryst (2012)



• Inverse isotope effect!

• $V(\text{H}_2\text{O}) < V(\text{D}_2\text{O})$ by 0.09%

• Deviation is increasing rather than decreasing as T increases

Should V_0 be
> expts. or < expts?



Theory of zero point and thermal lattice expansion (I)



- To find V at $T=0$, we minimize:

$$E_{gs} = E_{el,0} + \sum_q \frac{\hbar\omega_q}{2} \leftarrow E_{zp}$$

- At $T>0$ the equilibrium V minimizes the Helmholtz free energy:

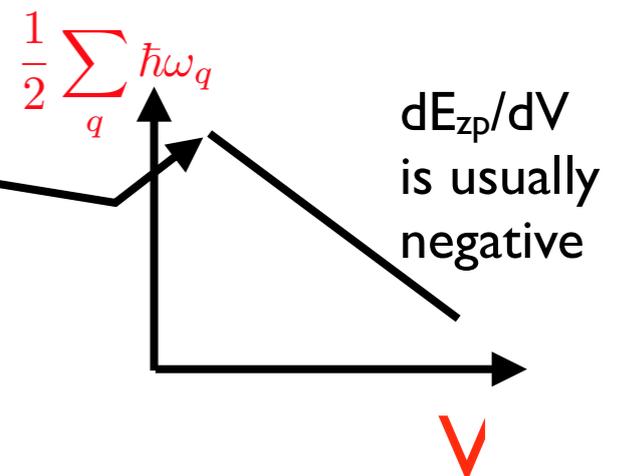
$$F(V, T) = E_0(V) + \sum_q \left[\frac{\hbar\omega_q}{2} + k_B T \ln \left(1 - e^{-\frac{\hbar\omega_q}{k_B T}} \right) \right]$$

- Volume dependent Quasiharmonic approximation: Frequencies depend only on $V(T)$.

$$\omega_q(V) = \omega_q(V_0) \left[1 - \frac{(V - V_0)}{V_0} \gamma_q \right]$$

- Grüneisen parameter

$$\gamma_q \equiv - \left(\frac{V_0}{\omega_q} \right) \left(\frac{\partial \omega_q}{\partial V} \right)_{V_0}$$





ICE STRUCTURES



Different Ice Structures

ice Ih (BF structure, proton ordered)

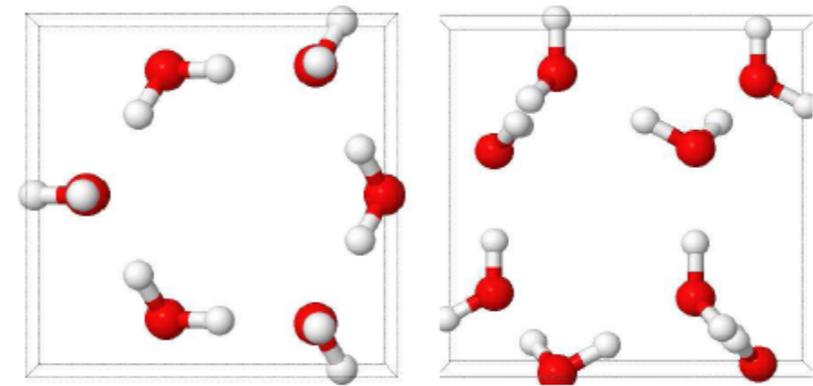


FIG. 2. Bernal-Fowler ice Ih structure. The image on the right is the top view of the x-y plane; the image on the left is the side view of the x-z plane.

ice XI

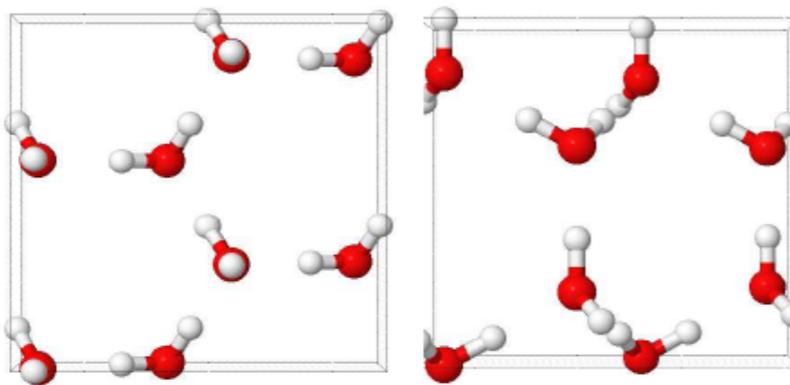


FIG. 1. Unit cell of the H-ordered ice XI structure. The image on the right is the top view of the x-y plane; the image on the left is the side view of the x-z plane.

ice Ih, 96 H₂O, proton disordered

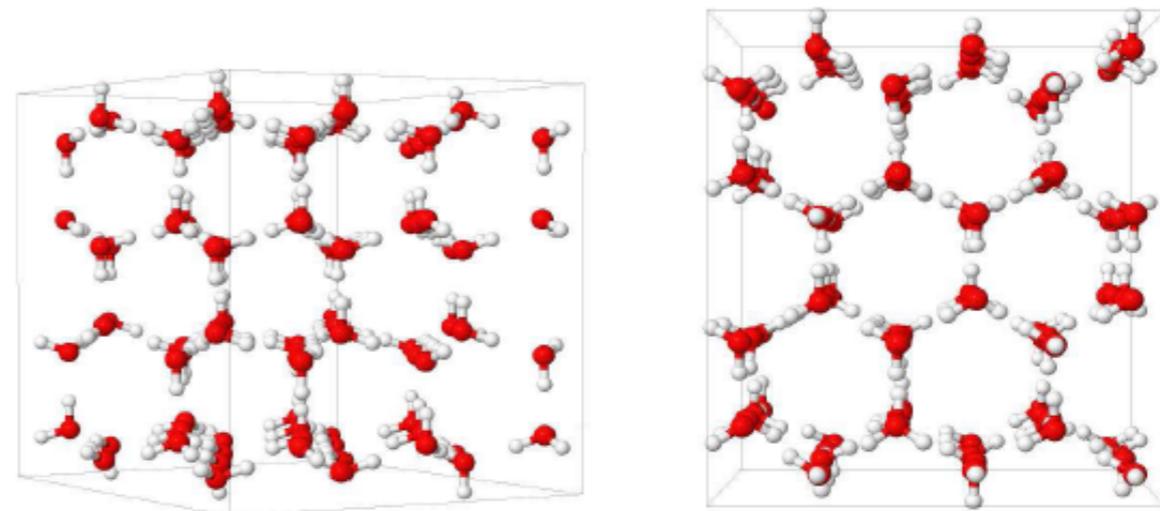
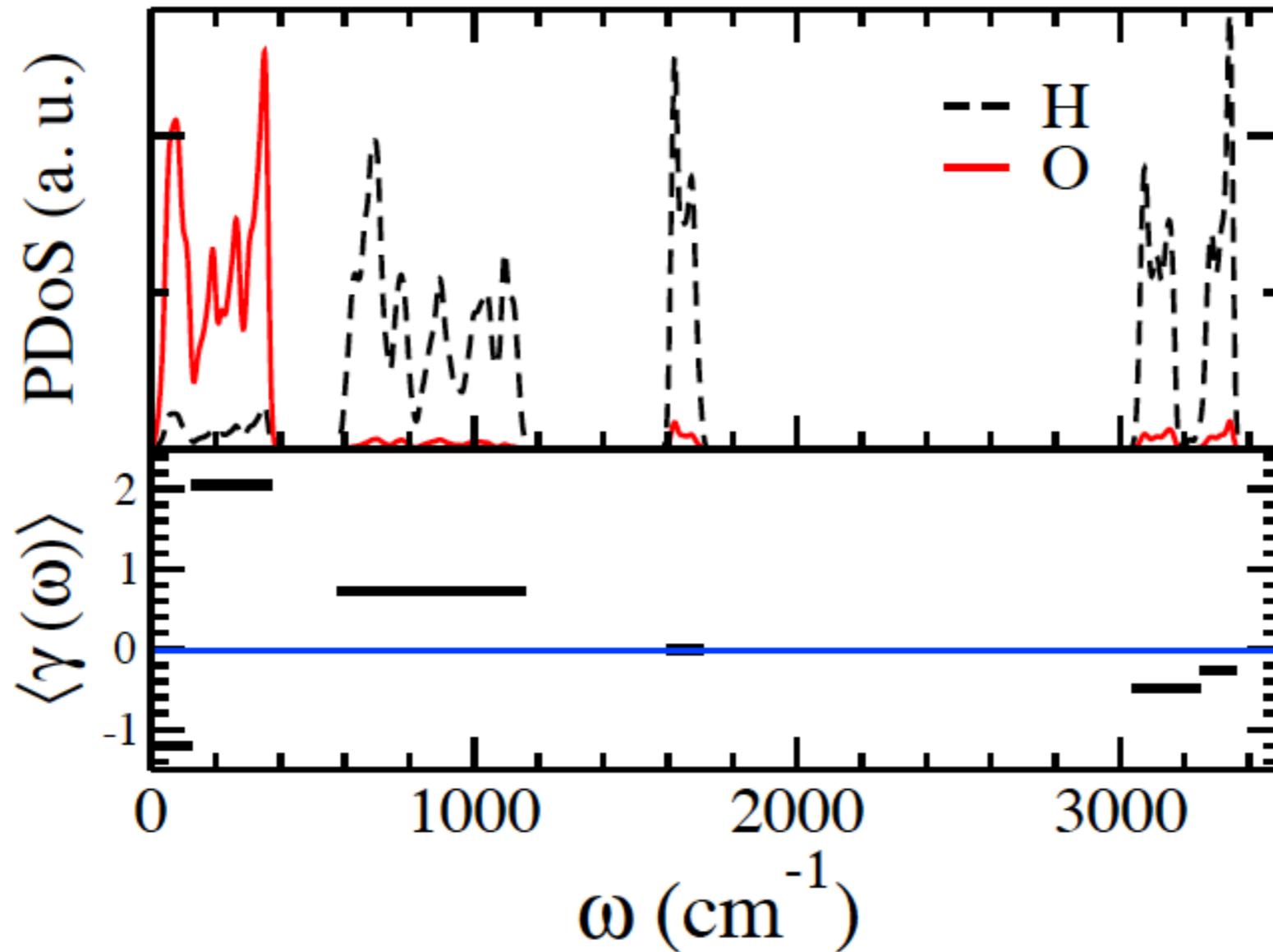


FIG. 3. H-disordered ice Ih structure. The image on the right is the top view of the x-y plane; the image on the left is the side view of the x-z plane.



Phonon Density of States



OH--O anti-correlation:
Stronger H-bond \Rightarrow
Weaker OH covalent bond

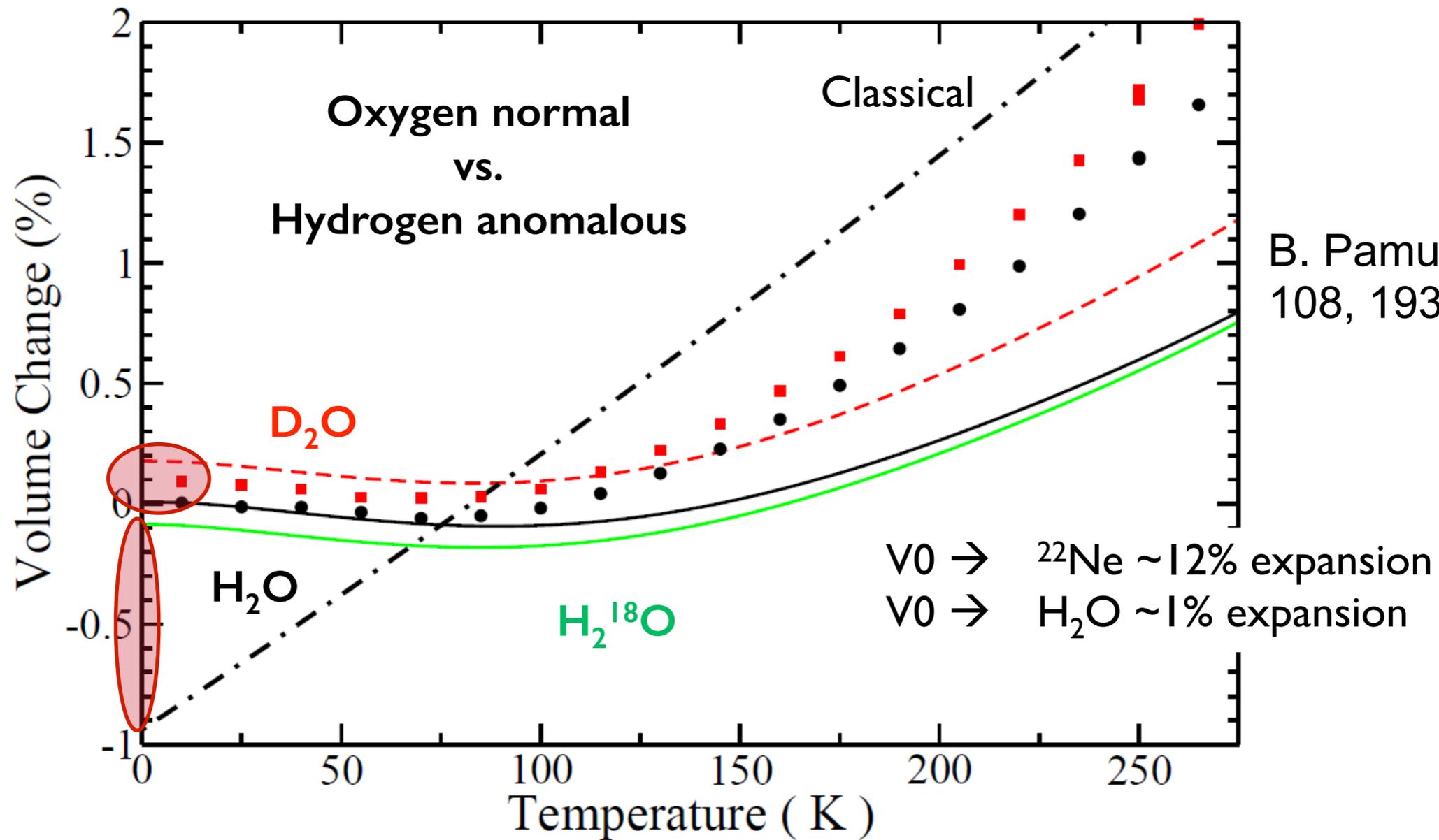
• Grüneisen parameter

$$\gamma_q \equiv - \left(\frac{V_0}{\omega_q} \right) \left(\frac{\partial \omega_q}{\partial V} \right)_{V_0}$$

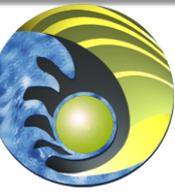
$\frac{1}{2} \sum_q \hbar \omega_q$
 dE_{zp}/dV is usually negative
 V



Temperature dependence: vdW-DF-PBE

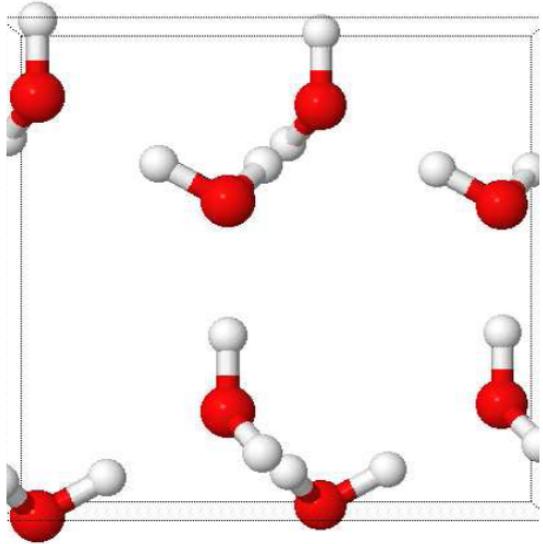


B. Pamuk et al, PRL
108, 193003 (2012)

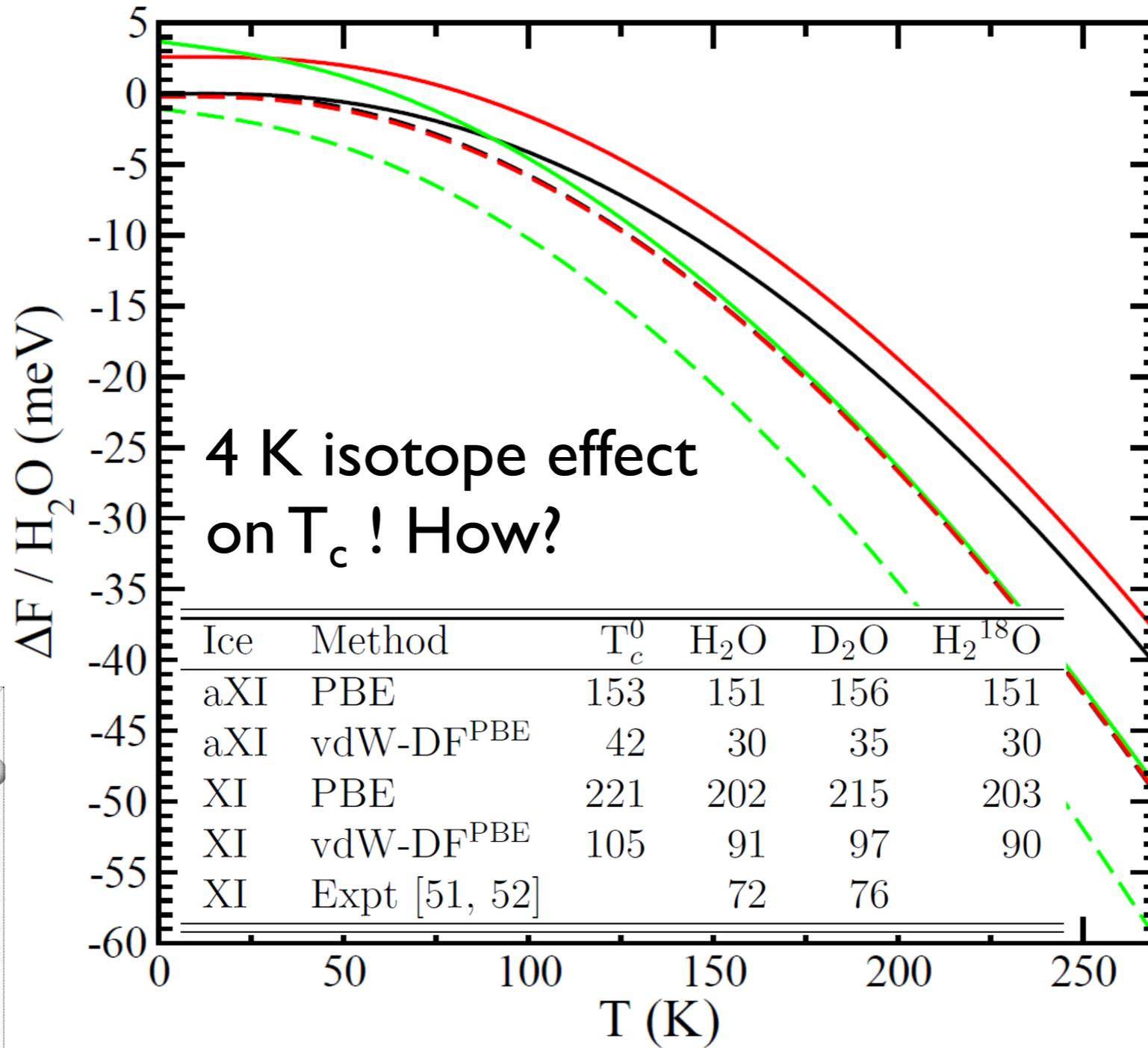
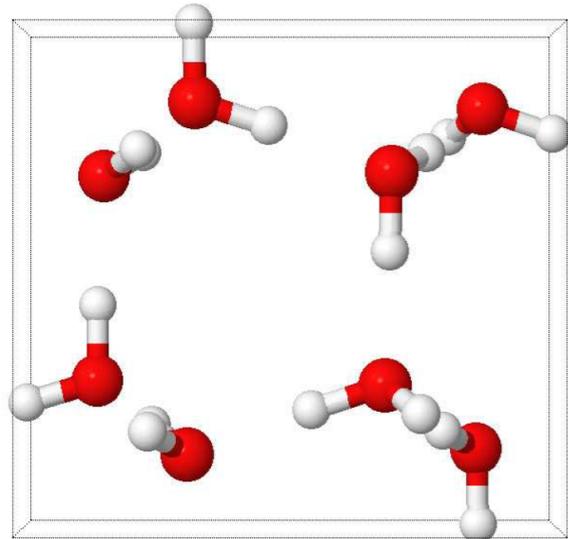


ICE I-XI ORDER DISORDER PHASE TRANSITION

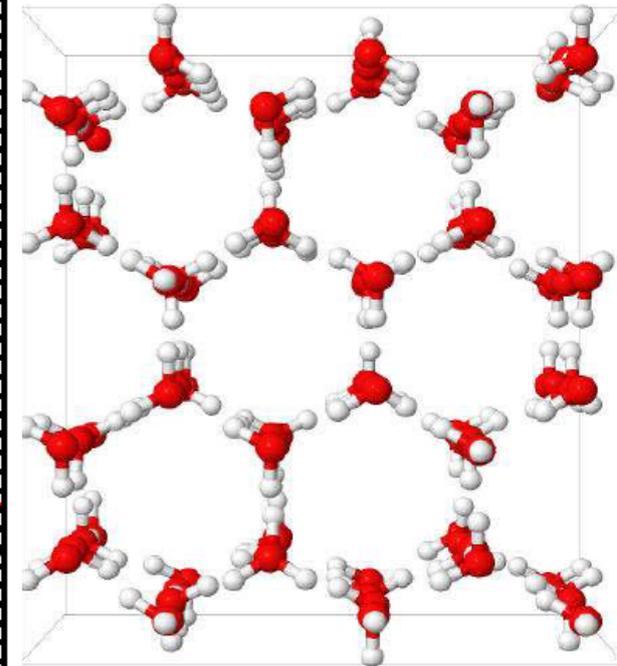
Ferroelectric
ordered ice XI



Antiferroelectric
ordered ice XI



Disordered
Ice Ih





ICE I-XI ORDER DISORDER PHASE TRANSITION

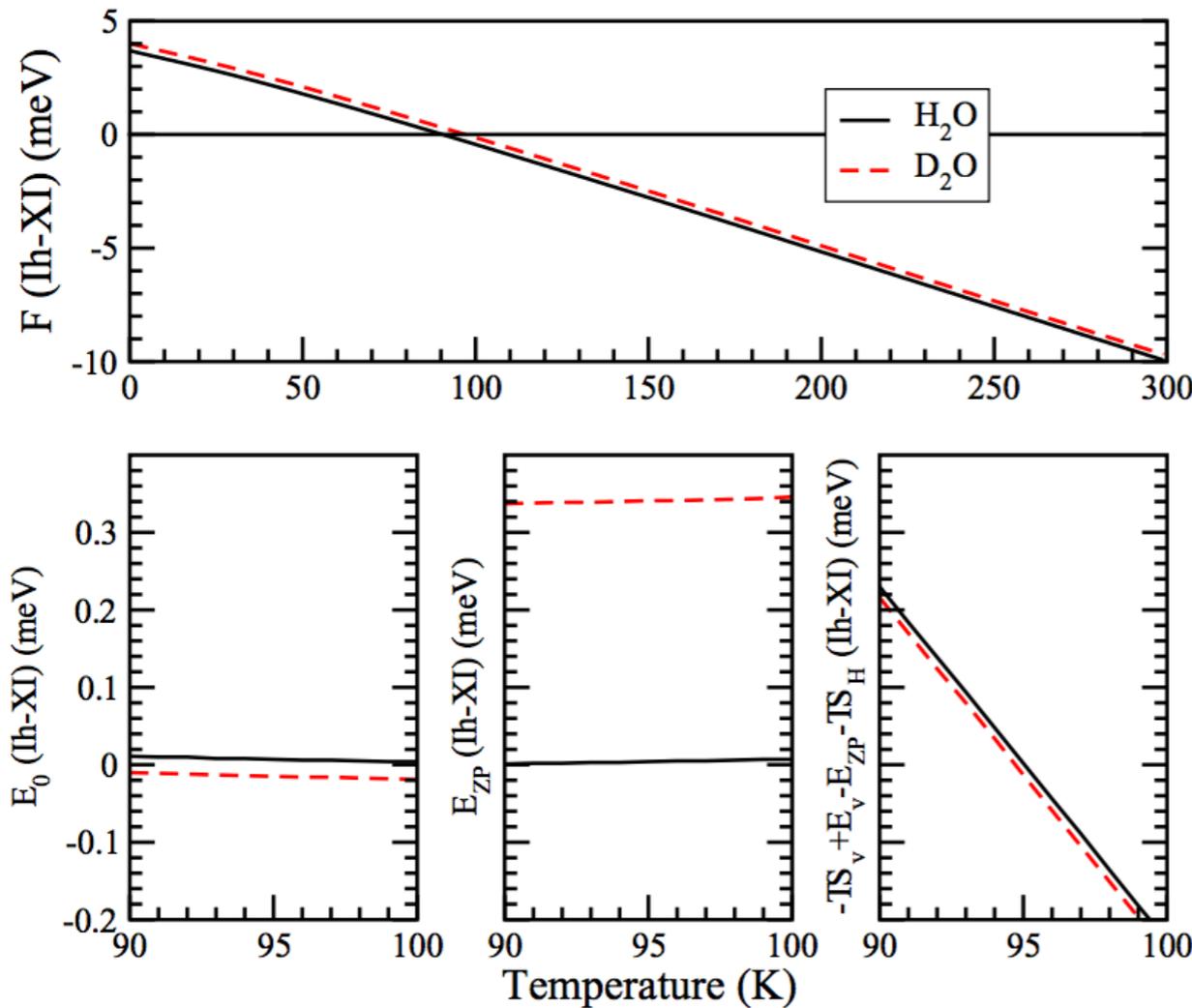


FIG. 5. (Color online) (Top) Free energy difference per molecule between ice Ih and ice XI calculated with vdW-DF^{PBE} functional in the region of the phase transition. (Bottom) Contributions to this free energy difference by each term in Eq. (4). (Left) Frozen lattice electronic term. (Middle) Zero-point vibrational energy. (Right) Remaining terms. All the energies on the bottom plots have been shifted to allow them to be compared in the same energy scale.

$$F(V, T) = E_0(V) + \sum_k \left[\frac{\hbar\omega_k(V)}{2} + k_B T \ln(1 - e^{-\hbar\omega_k(V)/k_B T}) \right] - TS_H. \quad (4)$$

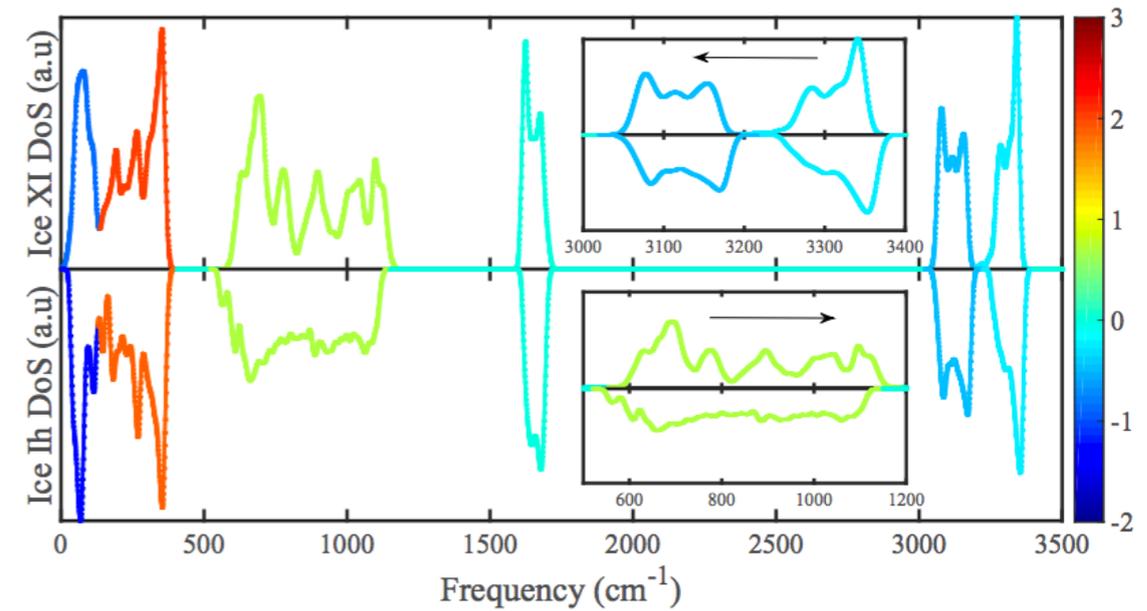


FIG. 6. (Color online) Vibrational density of states for H₂O for proton-ordered ice XI and disordered Ih structures, as obtained with vdW-DF^{PBE} functional. Average Grüneisen constants of the different modes are given in color code. The inset above zooms into the stretching modes and shows the redshift, while the inset below zooms into the librational modes and shows the blueshift in ice XI with respect to ice Ih.



Conclusions



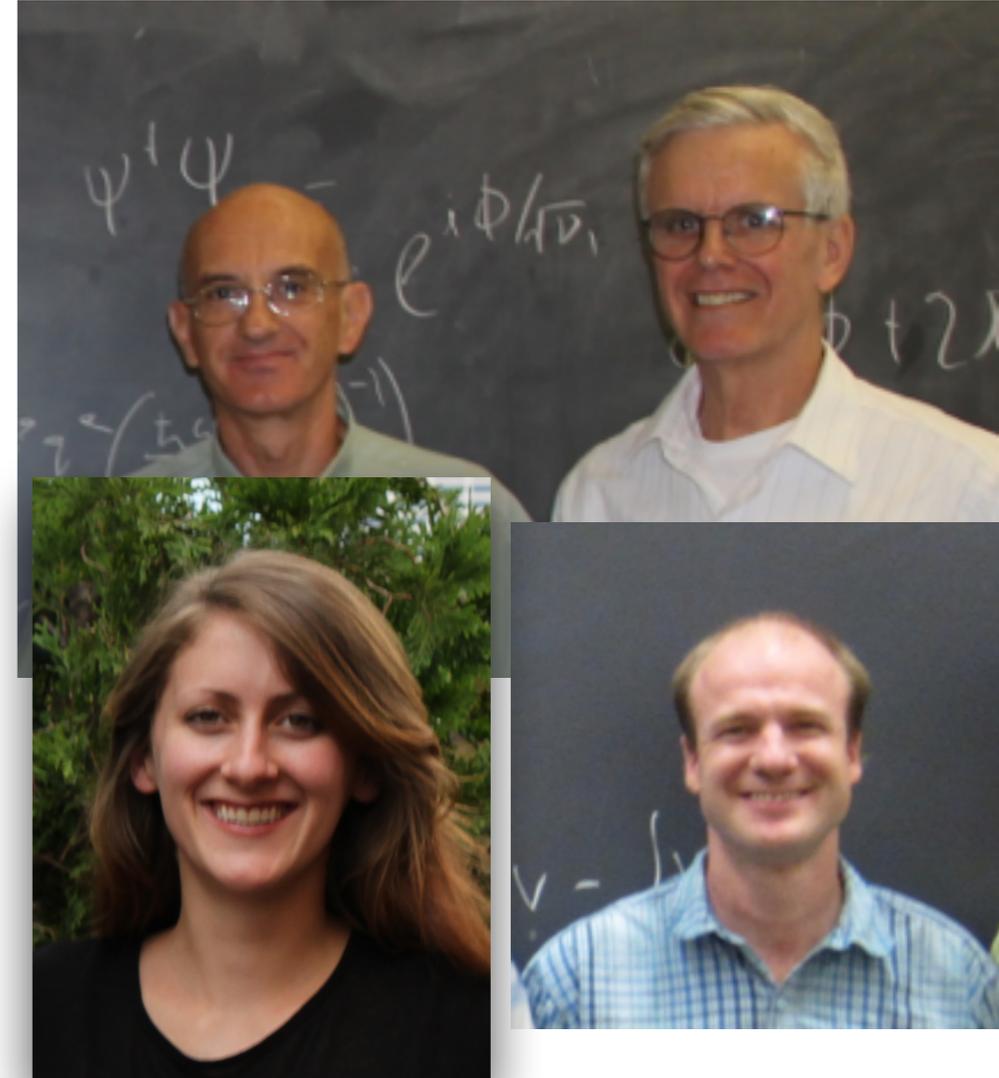
- The physics of water is rich. A book on condensed matter physics could be written using mostly water examples
 - phonons, thermal conductivity, dielectric properties, thermal expansion, phase transitions, ferroelectricity...
- In computational condensed matter physics water challenges us to develop new simulation methods and testing current methods
 - New density functionals, new molecular dynamics algorithms, simulation of nuclear quantum effects, efficient and accurate sampling.
- Lots of progress achieved in the last 10 years, and much more to come!



ACKNOWLEDGMENTS



Closer Collaborators



DOE Early Career DE-SC0003871
DOE: DE-FG02-09ER16052





DIPOLE LONG RANGE ORIENTATIONAL ORDER

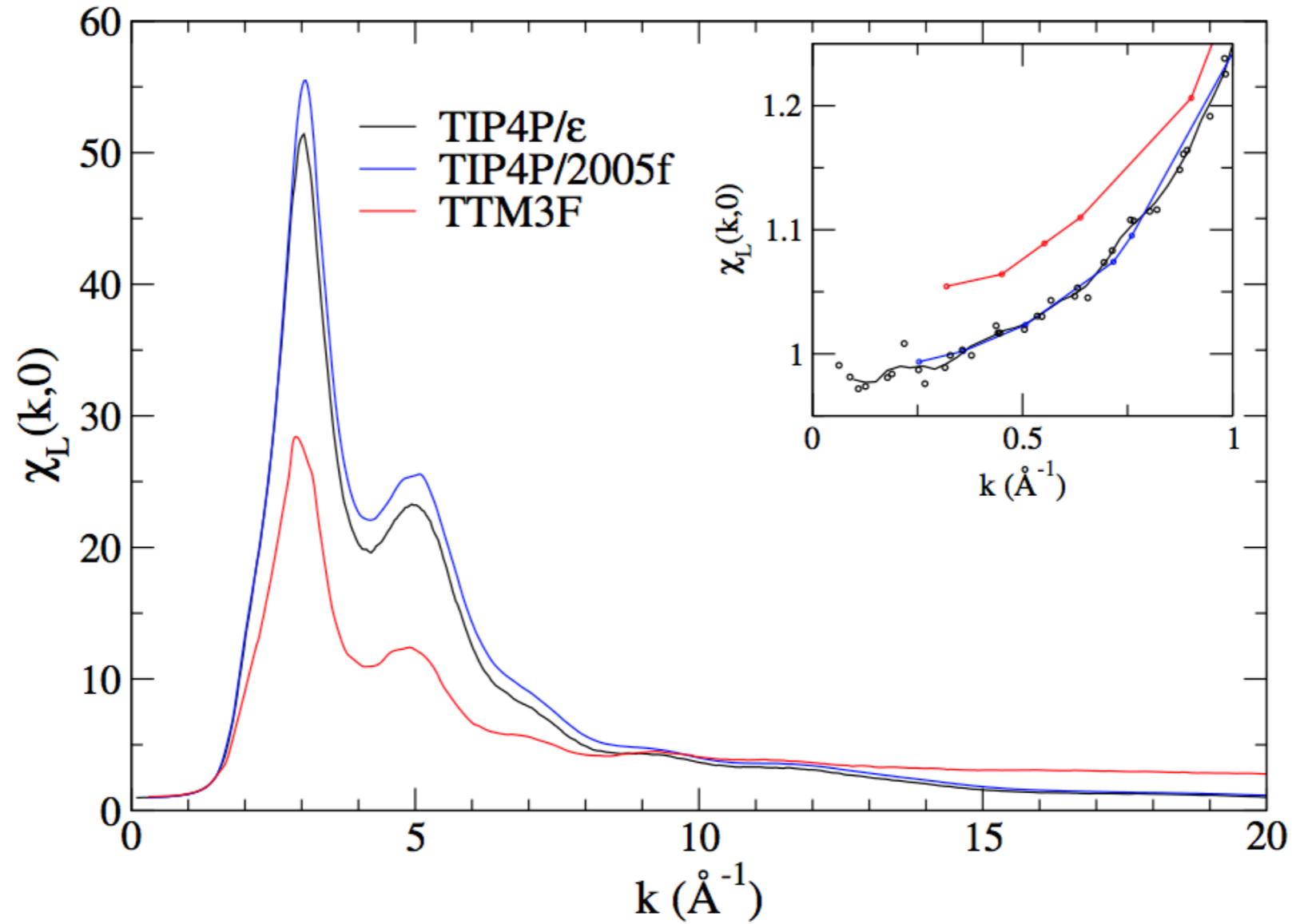


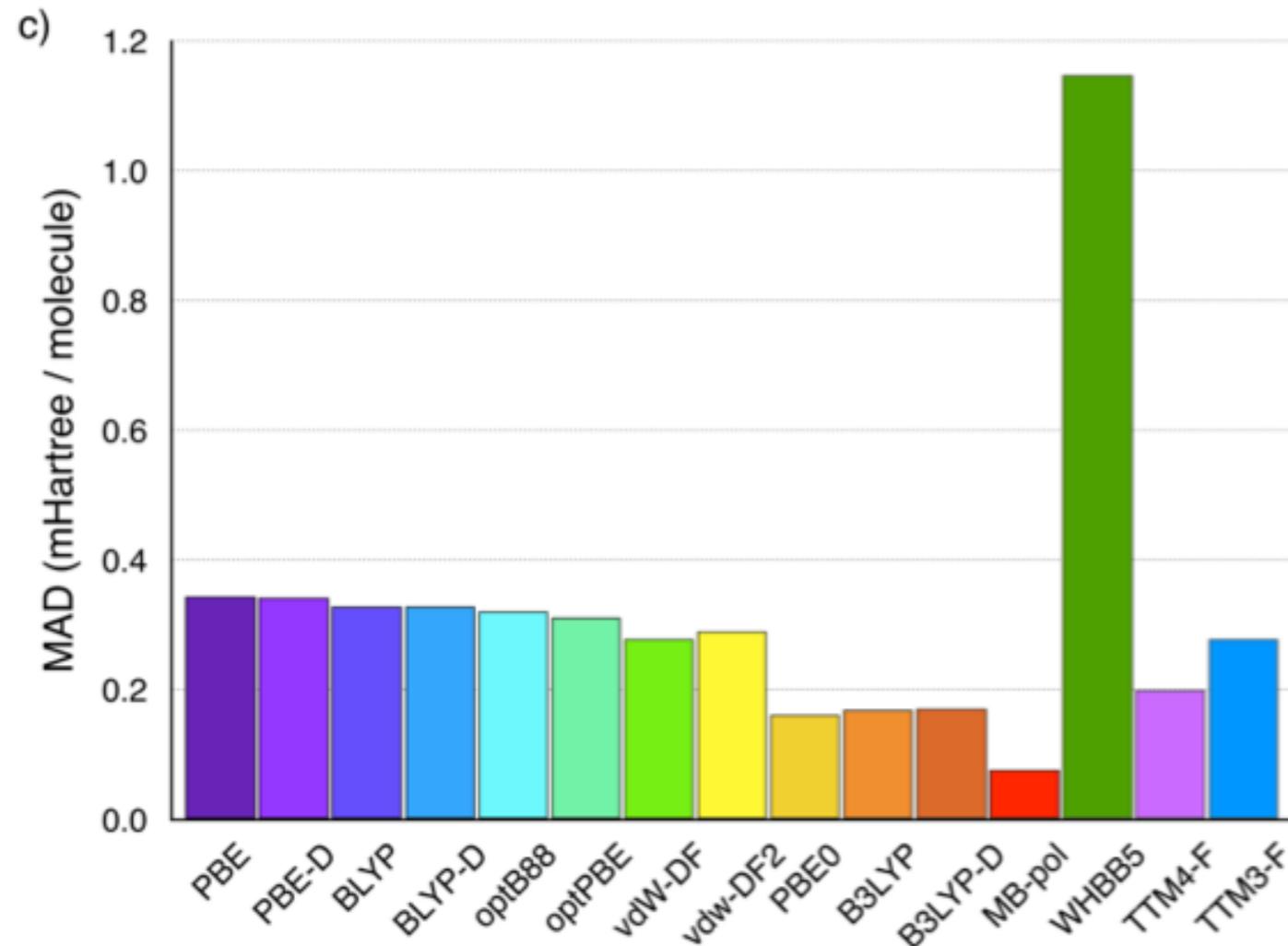
Figure 9.4: Static longitudinal susceptibility for three different models.



STATE OF THE ART IN THE WATER SIMULATION COMMUNITY: MB-POL

MB-POL: 2 and 3 body interactions at CCSD(T) level.

- V. Babin, C. Leforestier, and F. Paesani, J. Chem. Theory Comput., 9, 5395 (2013).
- V. Babin, G. R. Medders, and F. Paesani, J. Chem. Theory Comput., 10, 1599 (2014).



Medders et al. 2015

- Derived from fits to CCSD(T) energies calculated for both water dimers and trimers in the complete basis set (CBS) limit.
- Many-body effects: polarization model employed by the TTM4-F potential.
- To date MB-pol is the only many-body potential that accurately predicted the properties of water from the gas to the condensed phase.
- 47 times more expensive than TIP4P



MOTIVATION TO OPTIMIZE A DFT



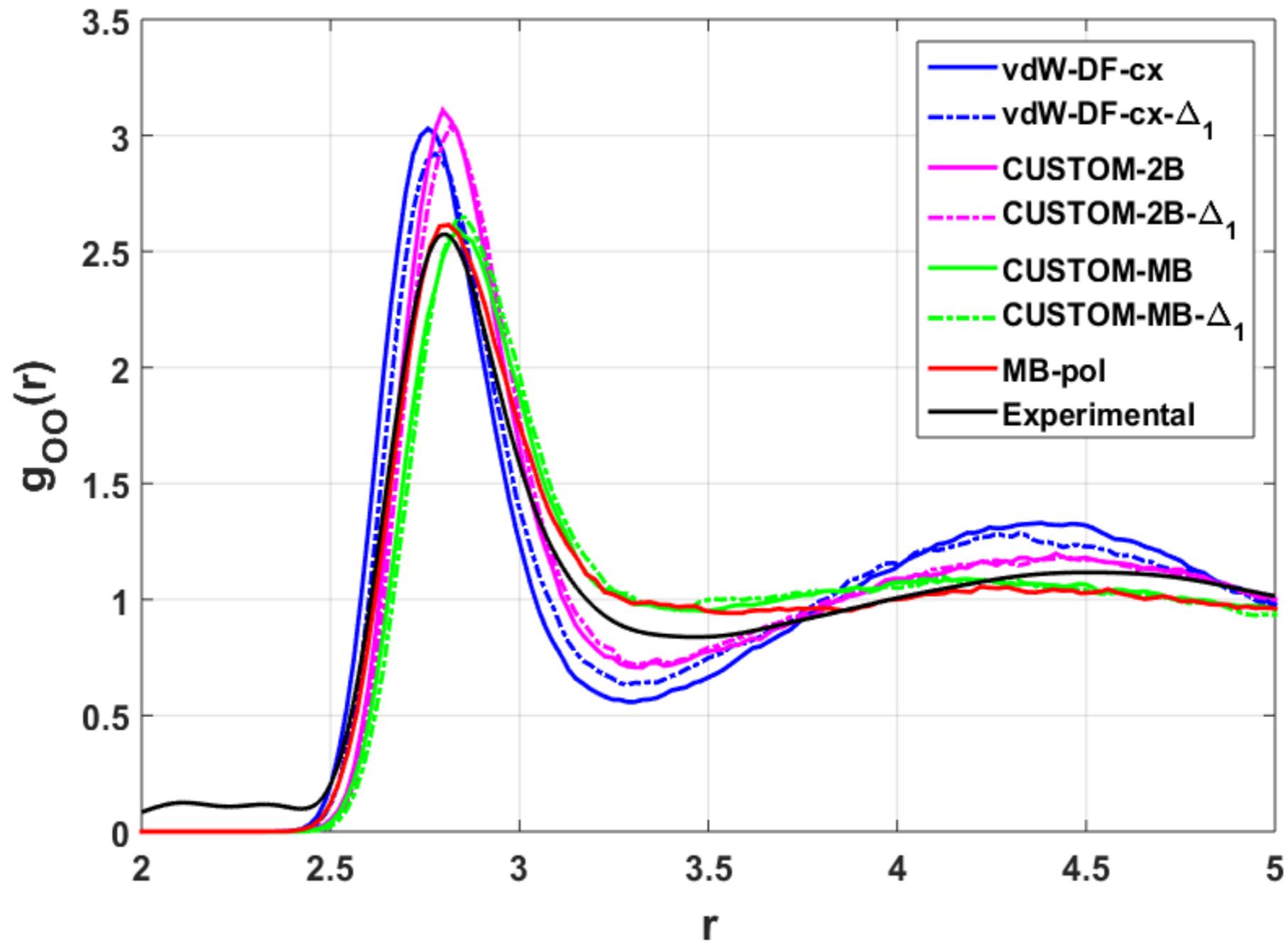
- **Can we optimize a density functional for water using the information produced to fit the MB-POL potential?**
- In practice, there is not one ab initio DFT, but many.
- Trial and error: does a functional fit the data?
- While hybrid functionals might work better, they are not yet as accurate as MB-pol and their computational cost is still too large.

We choose to take the MB-pol approach, and fit a GGA +vdW-Df functional. The method is called Data Projection on The parameter Subspace, DPPS.

DPPS: what functional fits best the data?

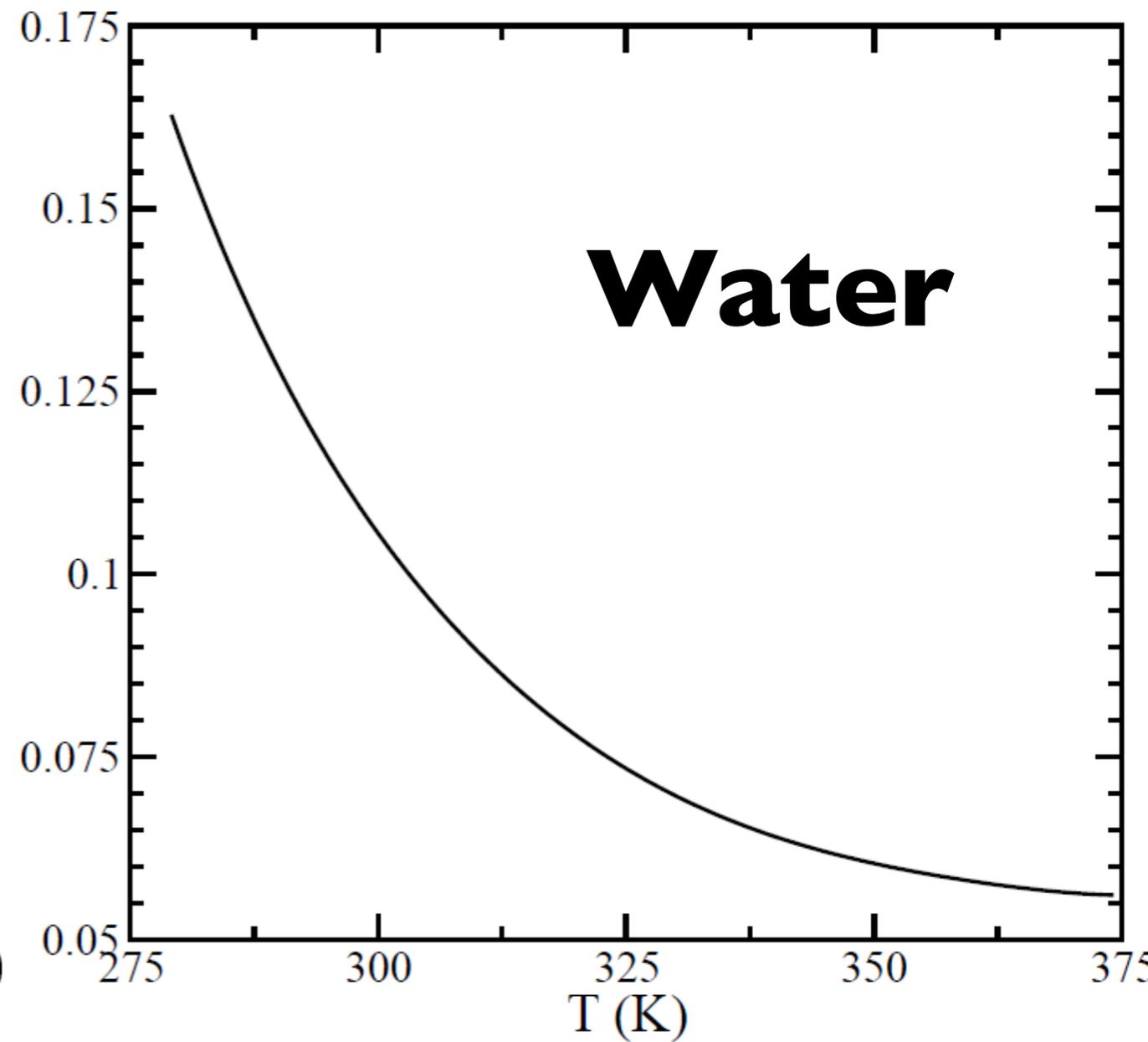
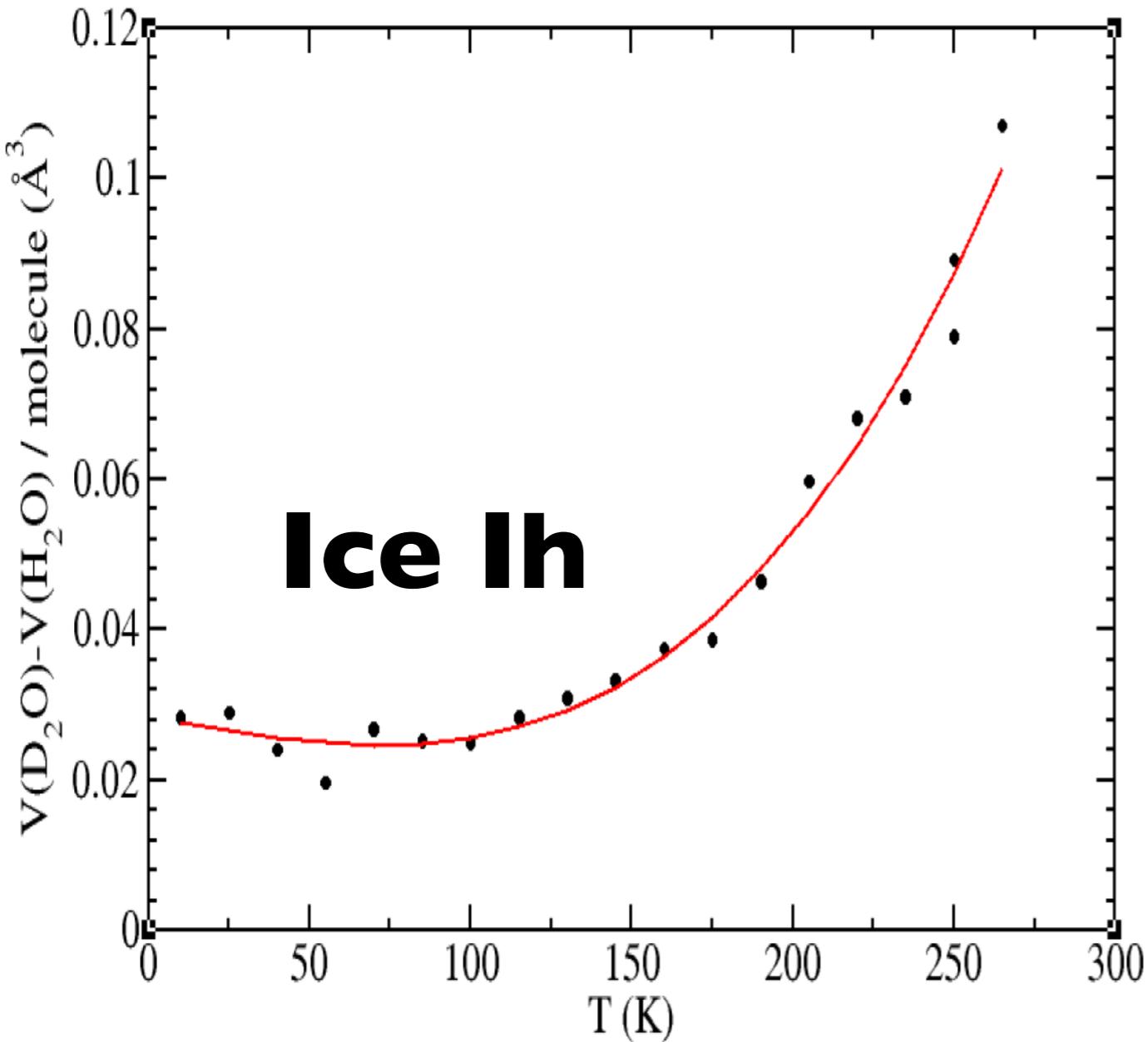


CUSTOM FUNCTIONAL: STRUCTURE





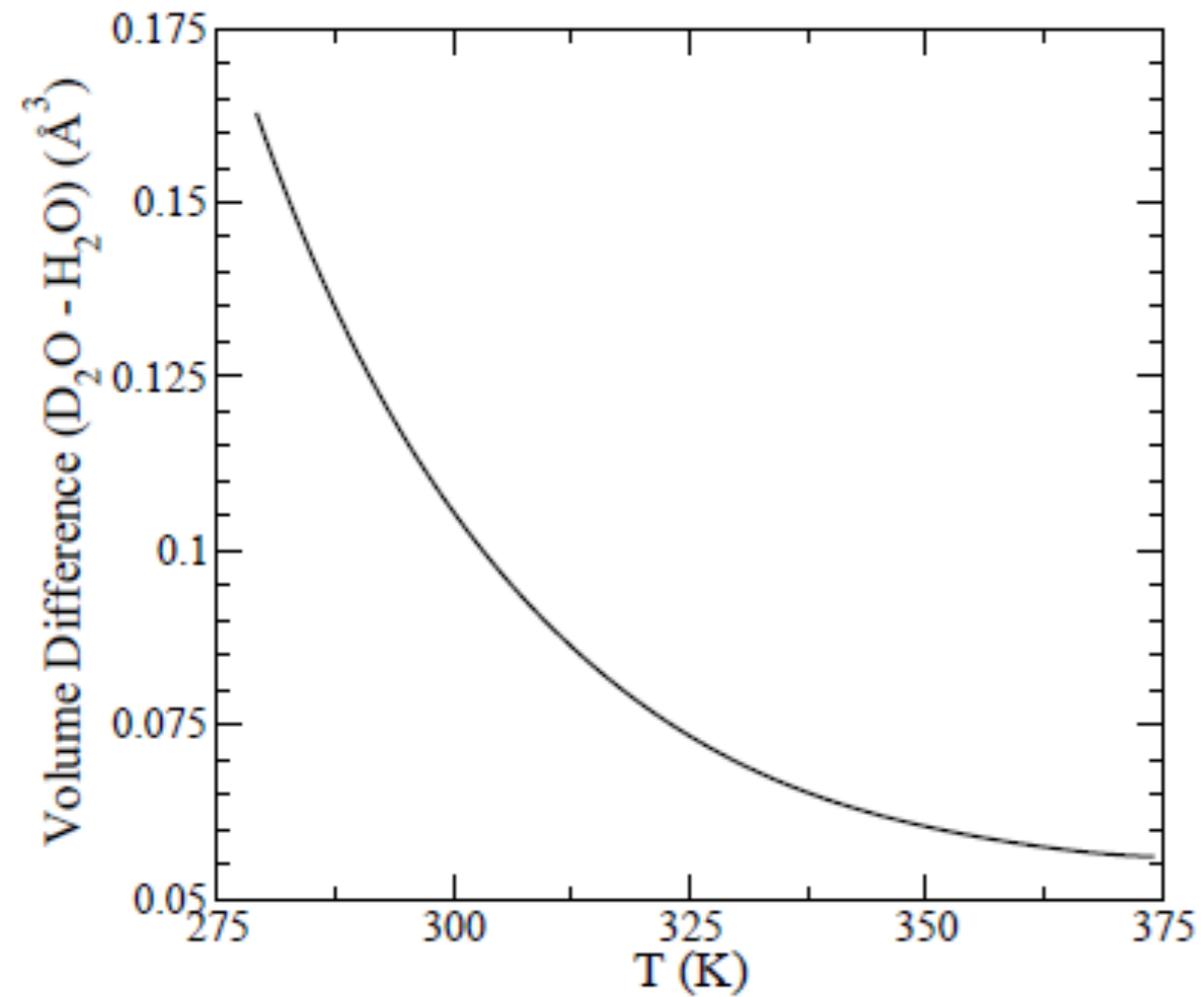
Anomalous Isotope Effect



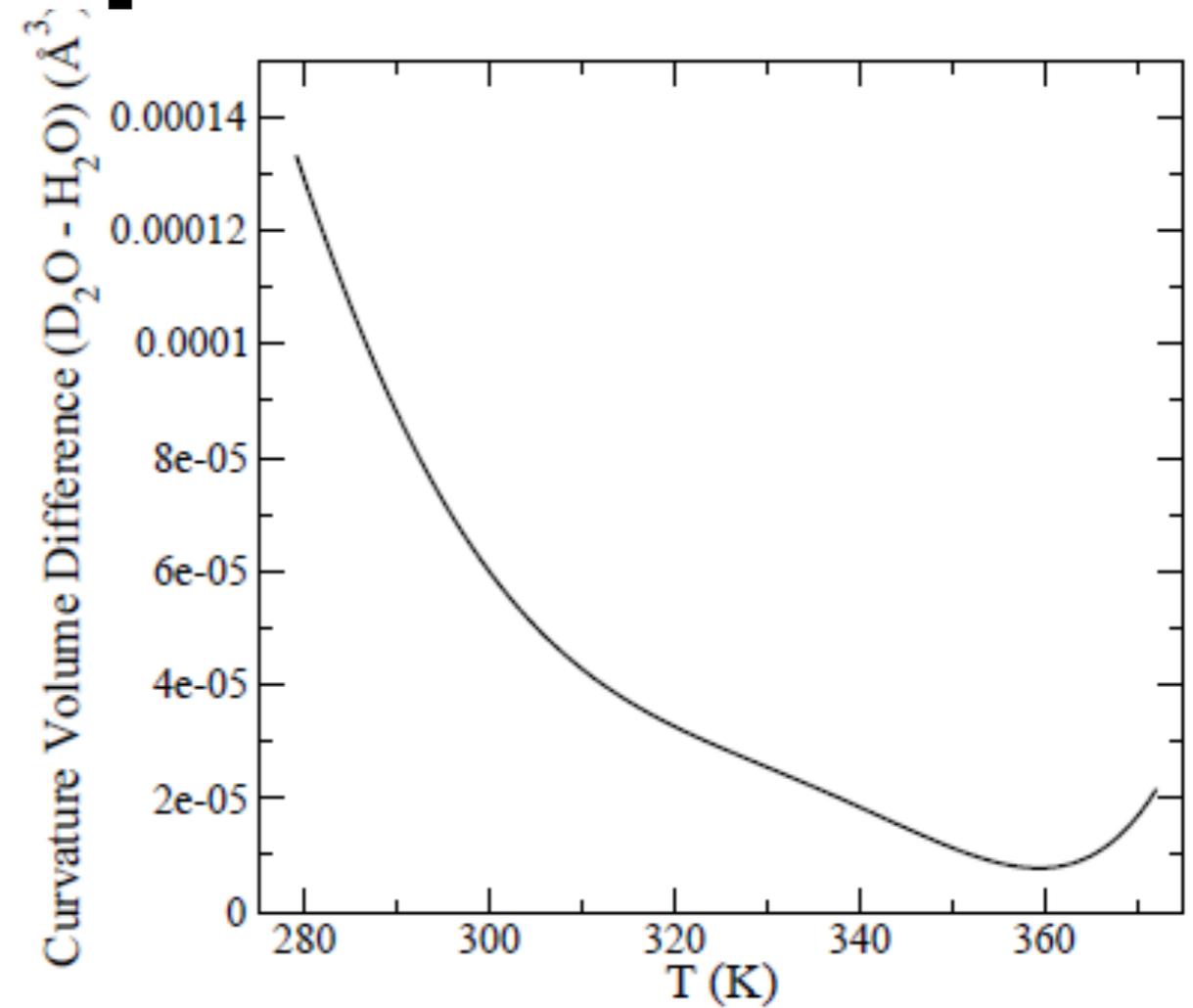
K. Röttger *et. al.*, *Acta Cryst.* **B50**, 644 (1994).
G. S. Kell, *J. Phys. Chem. Ref. Data* **6**, 1109 (1977).



From Ices to Liquid Water

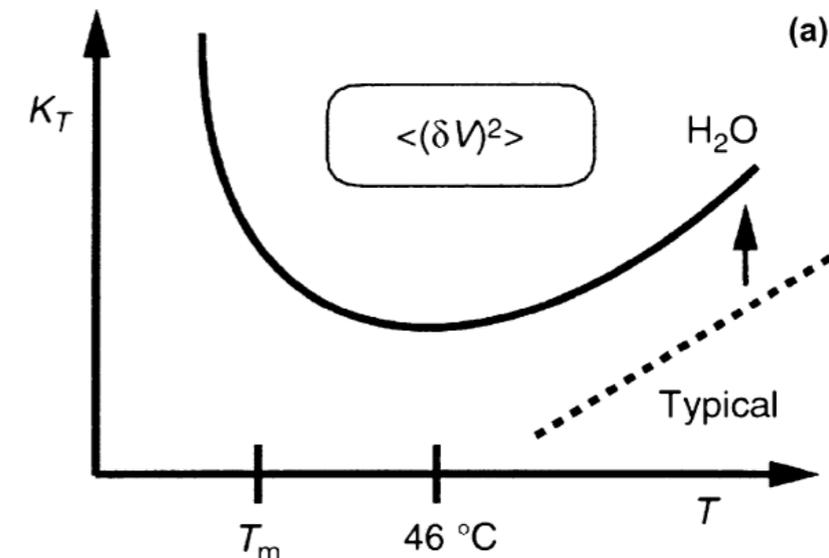


(a) Isotope effect in water.



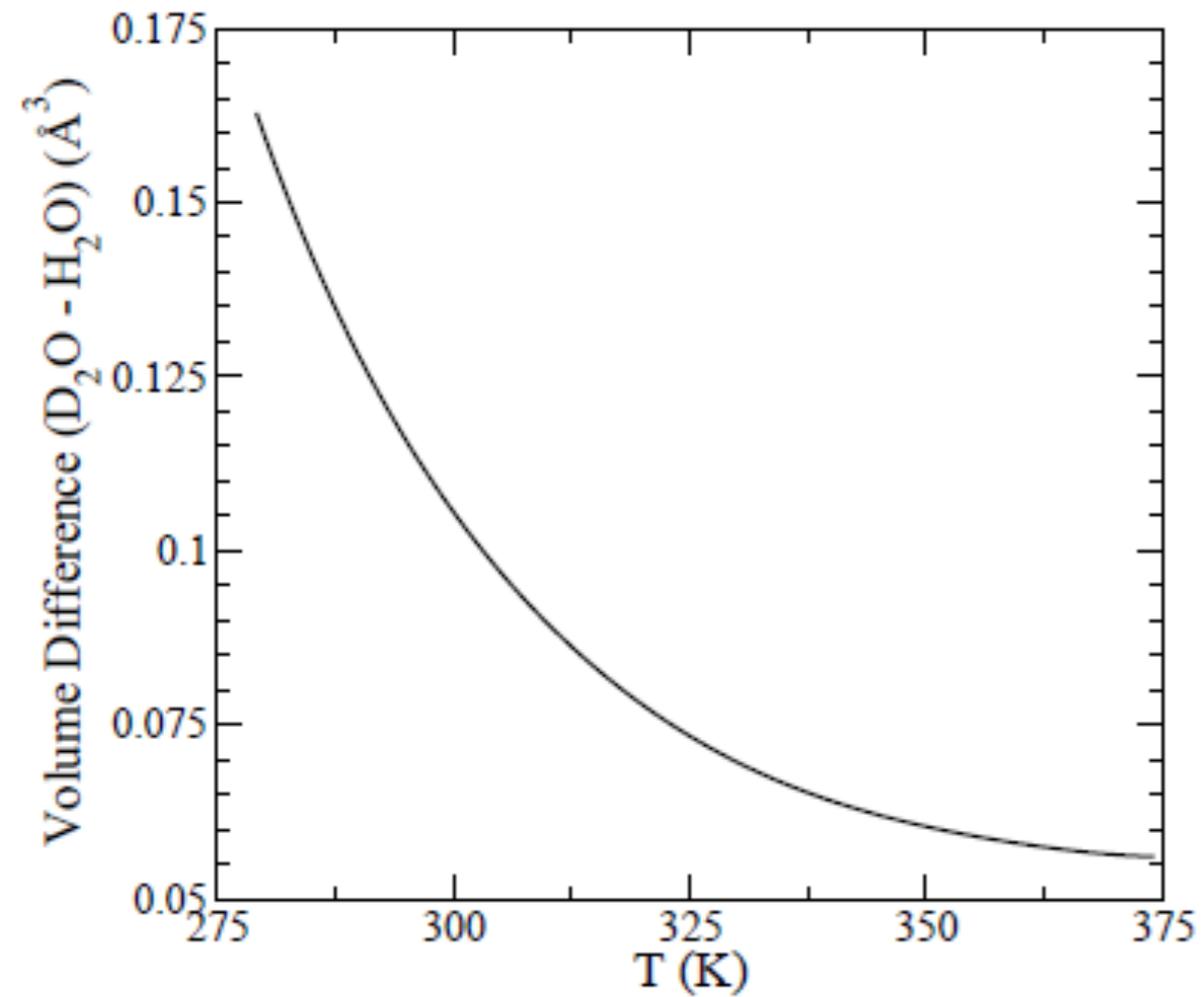
(b) Curvature of isotope effect in water.

Isothermal compressibility

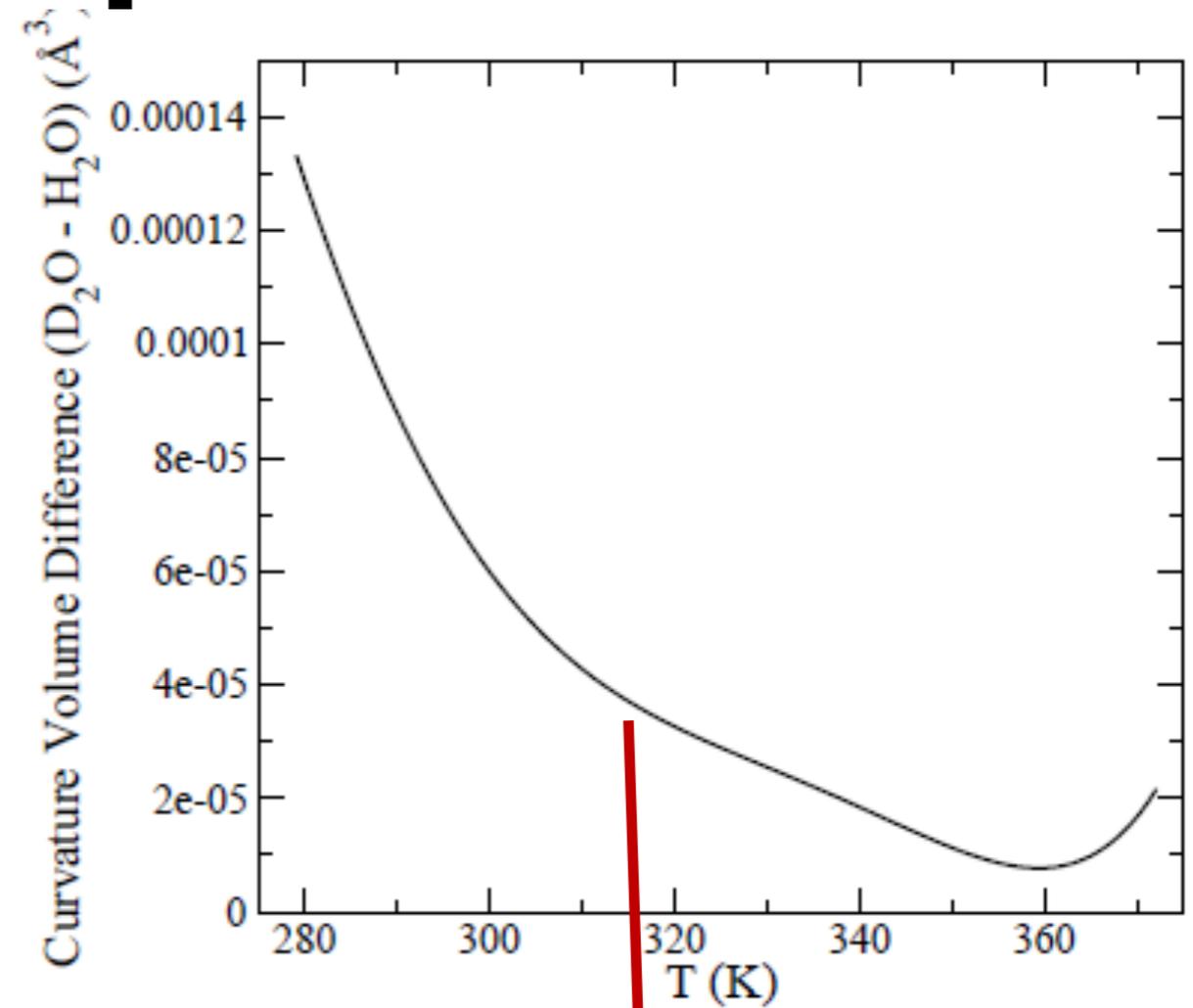




From Ices to Liquid Water



(a) Isotope effect in water.



(b) Curvature of isotope effect in water.

Isothermal compressibility

